

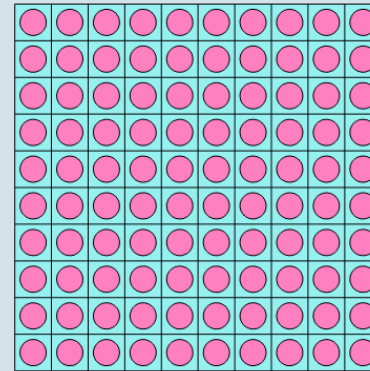
# COMSOL Minicourse: Material Homogenization



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# What is Homogenization?

- Replace a heterogeneous material subvolume by an *equivalent homogeneous material*
- Goal: find effective material properties
  - Young's modulus, Poisson's ratio
  - Coefficient of thermal expansion
  - Thermal conductivity
  - Permittivity, permeability
  - Coupling properties



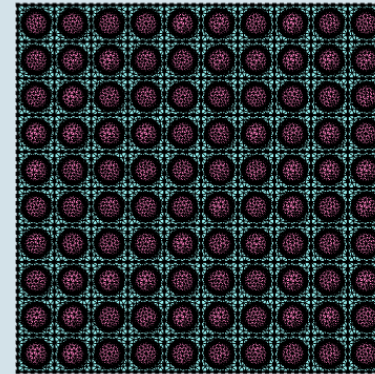
Heterogeneous  
microstructure



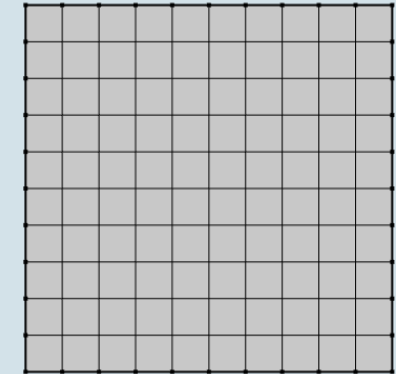
*Homogeneous equivalent*

# What is Homogenization?

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- Goal: find effective material properties
  - Young's modulus, Poisson's ratio
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  - Thermal conductivity
  - Permittivity, permeability
  - Coupling properties
- Fewer degrees of freedom, regular mesh



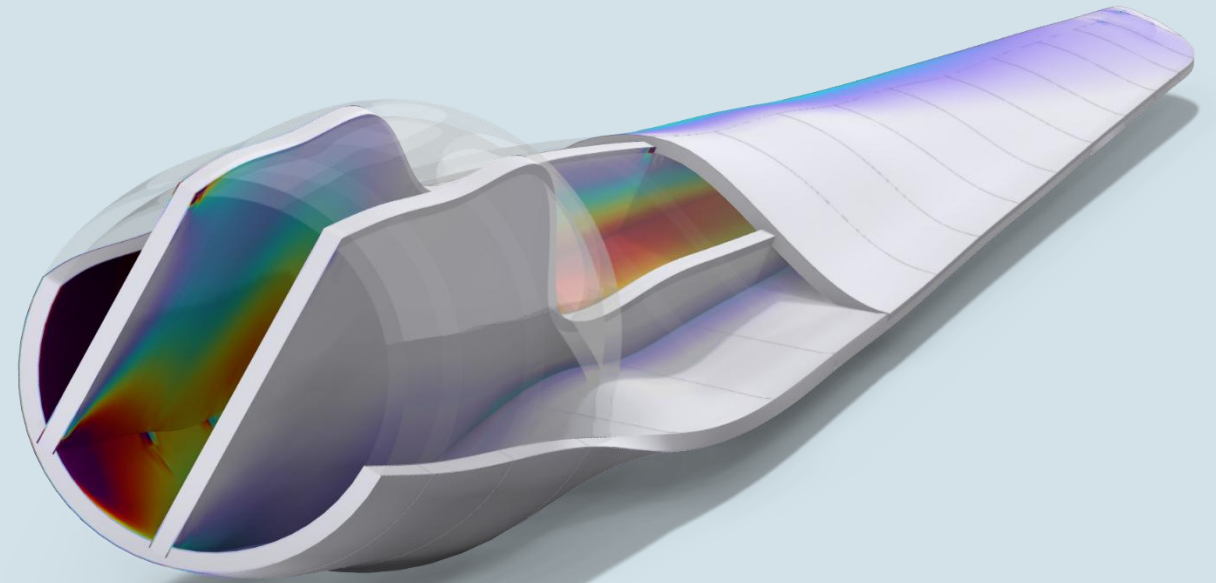
Heterogeneous  
microstructure



*Homogeneous equivalent*

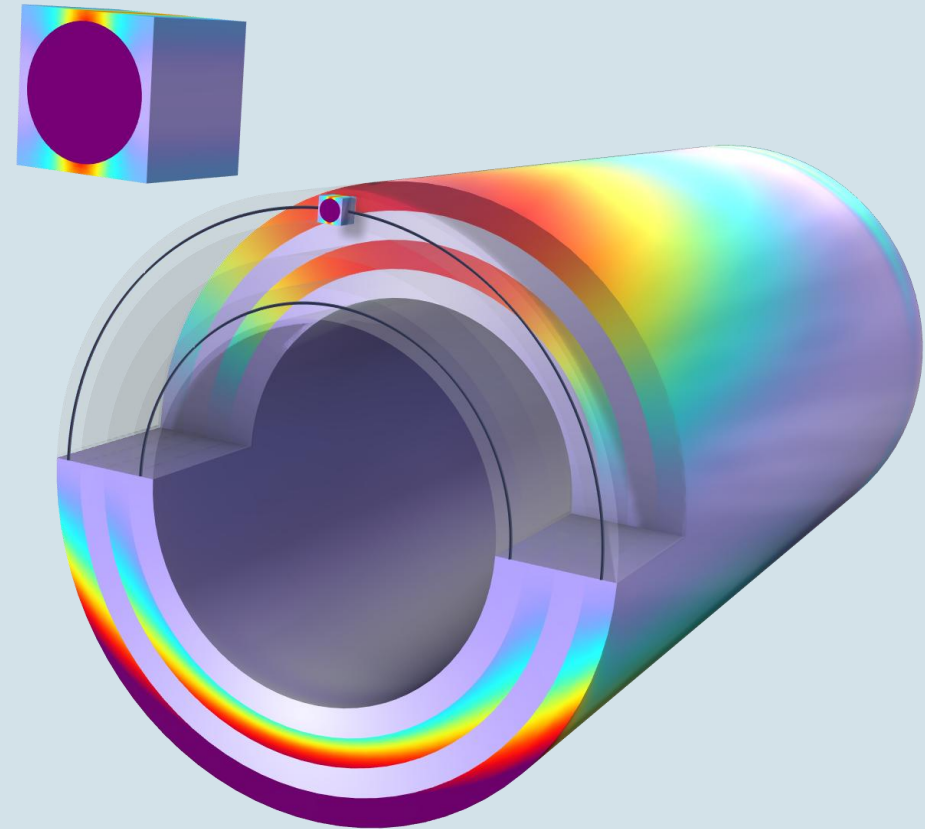
# When to Use Homogenization?

- Composite structures
  - Fibers, particles, weaves, laminates
  - Wind turbines, aerospace, automotive
- Lattice and cellular structures
  - Additive manufacturing
  - Lightweight design, metamaterials
- Incomplete material data
  - Generate anisotropic data from microstructure
  - Replace expensive measurements



# From Micro to Macro

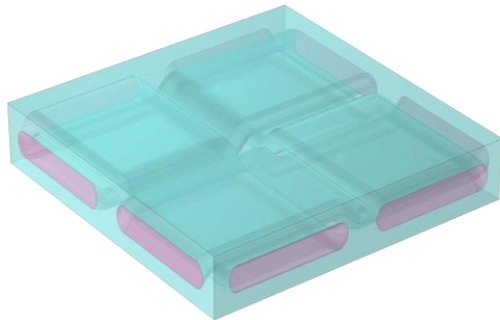
- Microscale
  - Repeating unit cell (RUC) or representative volume element (RVE)
  - Periodic or homogeneous boundary conditions
  - Output: effective material matrix
  - Analytical alternatives available
- Macroscale
  - Preferred technique for studying composite structures at global scales
  - Full part – tube, blade, panel
  - Behavior of composite structures based on homogenized materials
  - Massive reduction in DOFs



# Repeating Unit Cell vs Representative Volume Element

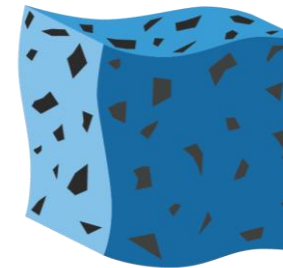
## Repeating Unit Cell - RUC

- Periodic geometry
- Examples: fiber composites, lattices, gyroids
- Periodic BC – exact result
- Size-independent – one cell is enough



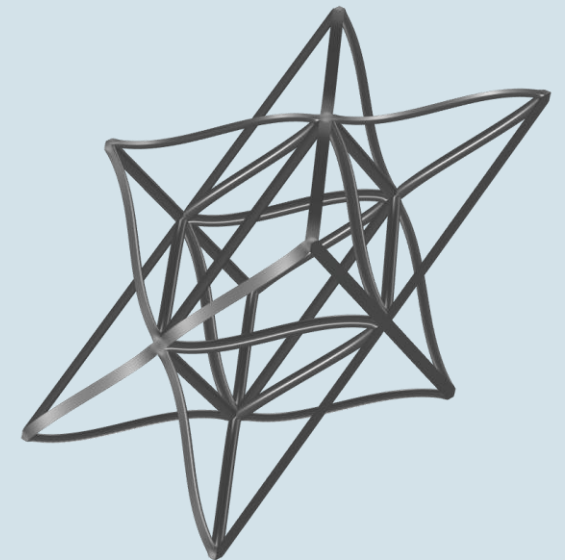
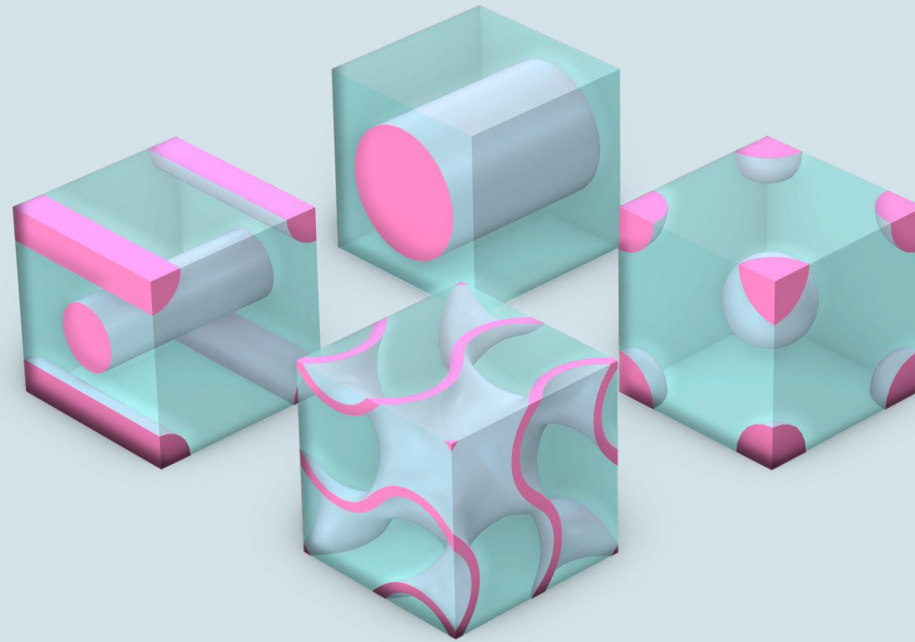
## Representative Volume Element - RVE

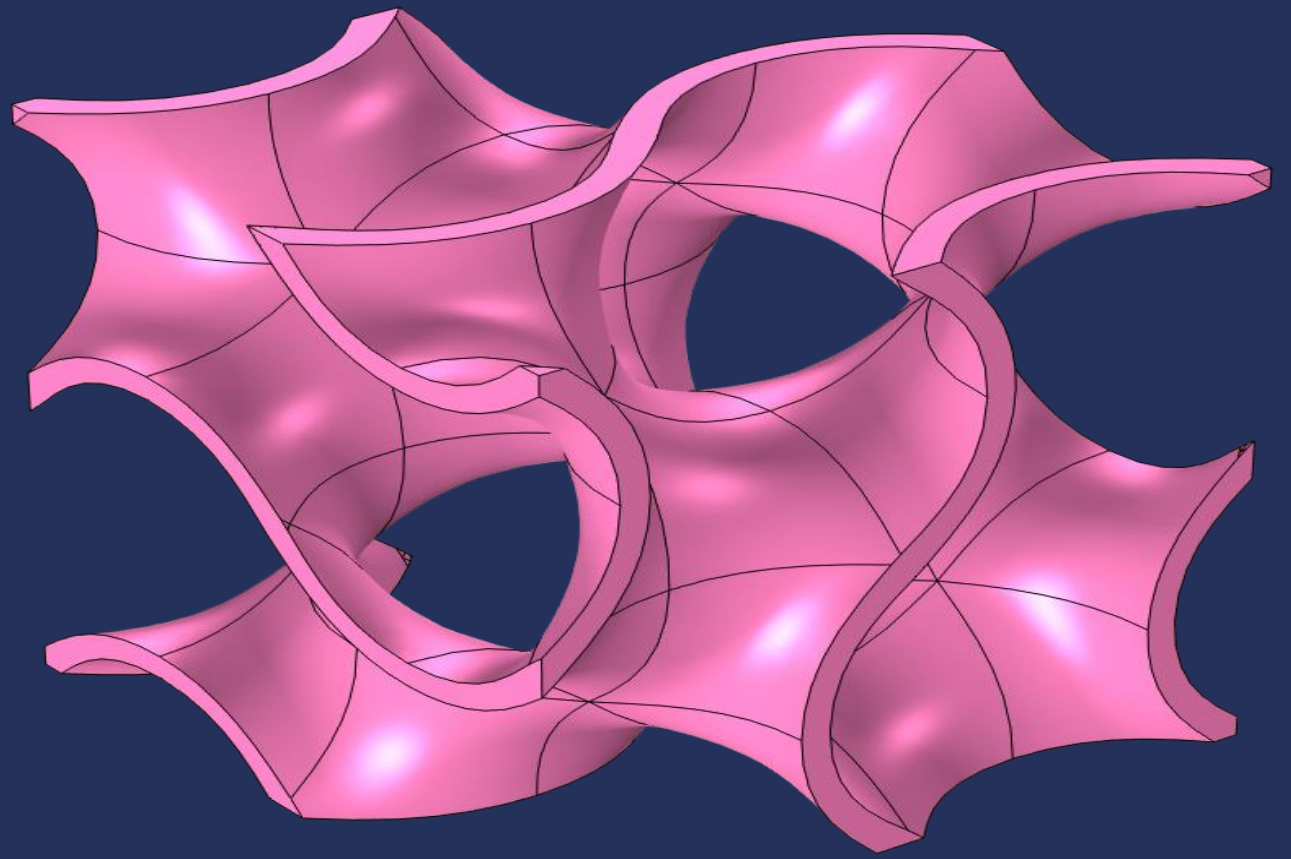
- Random microstructure – statistical sample
- Examples: random particulates, foams, batteries
- Homogeneous BC – displacement or traction
- Size dependent



# Material Homogenization in COMSOL®

- Cell Periodicity
  - Solid Mechanics, Heat Transfer, and MEMS
  - Effective stiffness, coefficient of thermal expansion, conductivity
- Part Library
  - Ready-to-use RUC and RVE geometries
  - Fiber composites, particulates, lattices, gyroids
- Random Particulate Composite RVE add-in

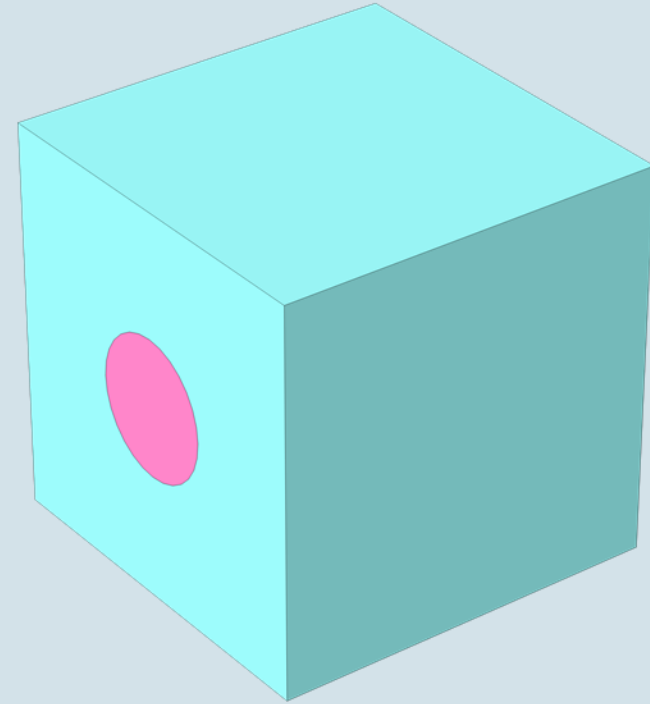




Showcase: Multiscale Modeling

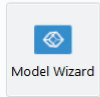
## Showcase Setup

- Microscale: Effective Properties
  - Unit cell (fiber + matrix) from Part Library
  - Solid Mechanics - Cell Periodicity
  - Periodic BCs → effective elasticity matrix
- Macroscale: Composite Cylinder
  - Layered Shell with Layered Material
  - Homogenized properties from microscale

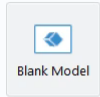


File

# New



Model Wizard



Blank Model



File Home Definitions Geometry Materials Physics Mesh Study Results Developer

Application Builder Model Manager Component 1 Add Component Parameters Functions Variable Utilities Equation Contributions Build All LiveLink Part Libraries Add Material Solid Mechanics Add Physics Add Mathematics Build Mesh Mesh 1 Compute Cell Periodicity Study Add Study Stress (solid) Add Plot Group Result Templates Windows Reset Desktop

### Model Builder

Type filter text

- Untitled.mph (root)
  - Global Definitions
    - Parameters 1
    - Geometry Parts
    - Materials
      - T-300 carbon fiber (mat1)
      - Epoxy polymer (mat2)
      - Homogeneous Material (solidcp1)
        - Basic (def)
        - Anisotropic (Anisotropic)
    - Load and Constraint Groups
      - Load Groups for Cell Periodicity
    - Component 1 (comp1)
      - Definitions
      - Geometry 1
        - Unidirectional Fiber Composite, S
          - Form Union (fin)
        - Materials
          - Material Link 1 (matlink1)
          - Material Link 2 (matlink2)
        - Solid Mechanics (solid)
          - Linear Elastic Material 1
            - Free 1
            - Initial Values 1
            - Cell Periodicity 1
              - Boundary Pair 1
              - Boundary Pair 2
              - Boundary Pair 3
          - Equation View
        - Mesh 1
          - Size
          - Identical Mesh 1
          - Free Triangular 1
          - Swept 1
        - Cell Periodicity Study
          - Step 1: Stationary
          - Solver Configurations
          - Job Configurations
        - Results
          - Datasets
          - Views
          - Derived Values
          - Tables

### Settings Properties

Anisotropic Name: Anisotropic

#### Output Properties

Property	Variable	Expression	Unit
Elasticity matrix	(D11, D...	{1.402647759360354...	Pa
Loss factor for elast...	eta_D_is...	0	1

Expression:

```
{1.4026477593603543E11, 2.1641862988512874E9, 8.172596112370869E9, 2.164186358867227E9, 1.5779349455417542E9, 8.172596471673254E9, -4.3028340434098984E-5, -9.149254596624857E-5, -1.2437232836837755E-4, 4.615024000236719E9, 32.45433205330848, -1556.1555693366713,
```

#### Model Inputs

Physical quantity	Variable
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#### Local Properties

Name	Expression	Unit	Description
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Name:

Expression:

### Graphics

z  
y x

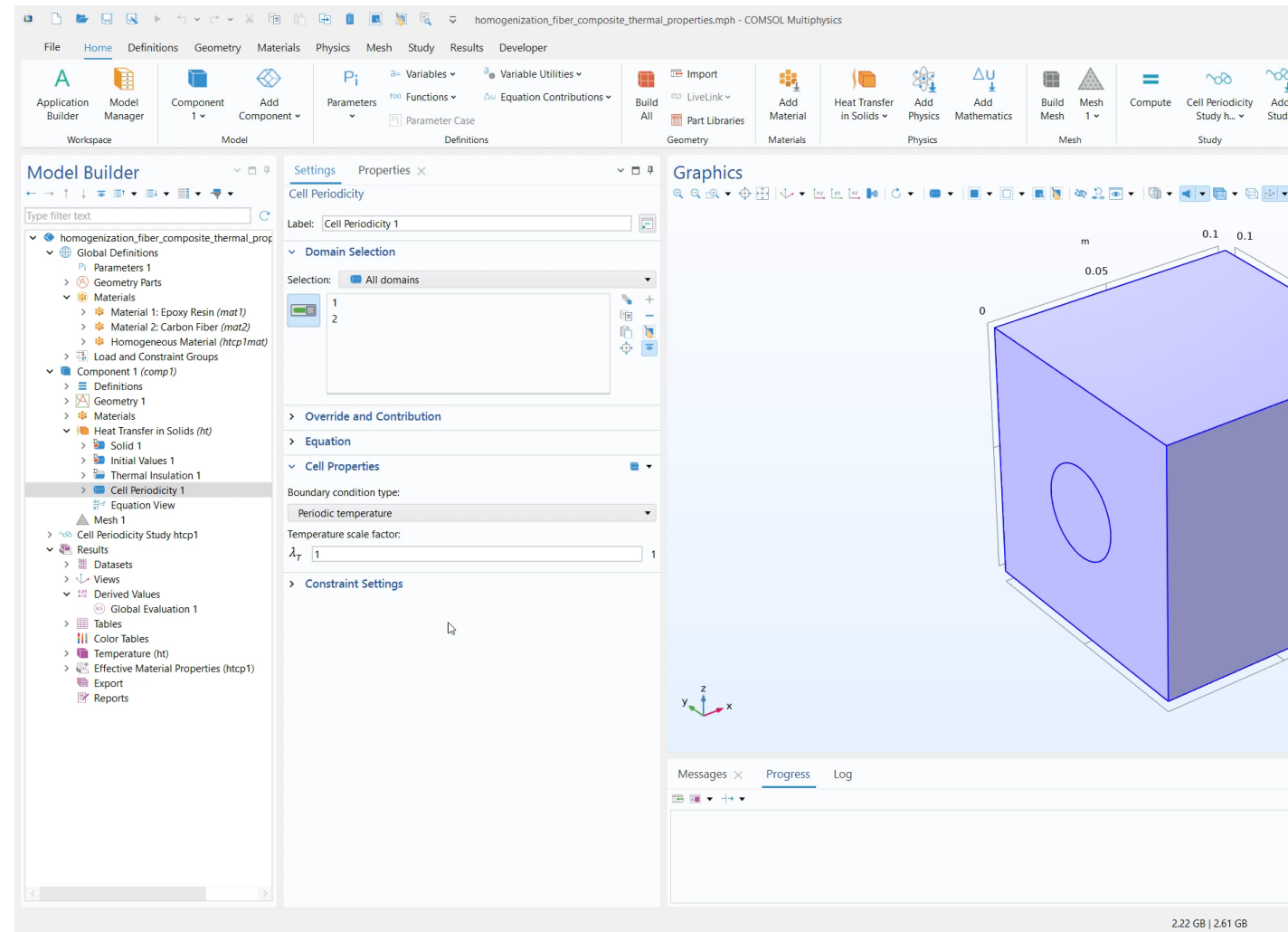
$\times 10^{-4} \text{ m}$

### Messages Progress Log Effective Material Properties (Cell P...

Density (kg/m^3)	Elasticity matrix (Pa)					
1556.0	1.4026E11	2.1642E9	2.1642E9	-4.3028E-5	32.454	-2.9213E-5
	2.1642E9	8.1726E9	1.5779E9	-9.1493E-5	-1556.2	-9.0913E-5
	2.1642E9	1.5779E9	8.1726E9	-1.2437E-4	1750.5	-9.9418E-5
	-4.3028E-5	-9.1493E-5	-1.2437E-4	4.6150E9	-2.6025E-4	-233.66
	32.454	-1556.2	1750.5	-2.6025E-4	2.6448E9	-3.1660E-5
	-2.9213E-5	-9.0913E-5	-9.9418E-5	-233.66	-3.1660E-5	4.6150E9

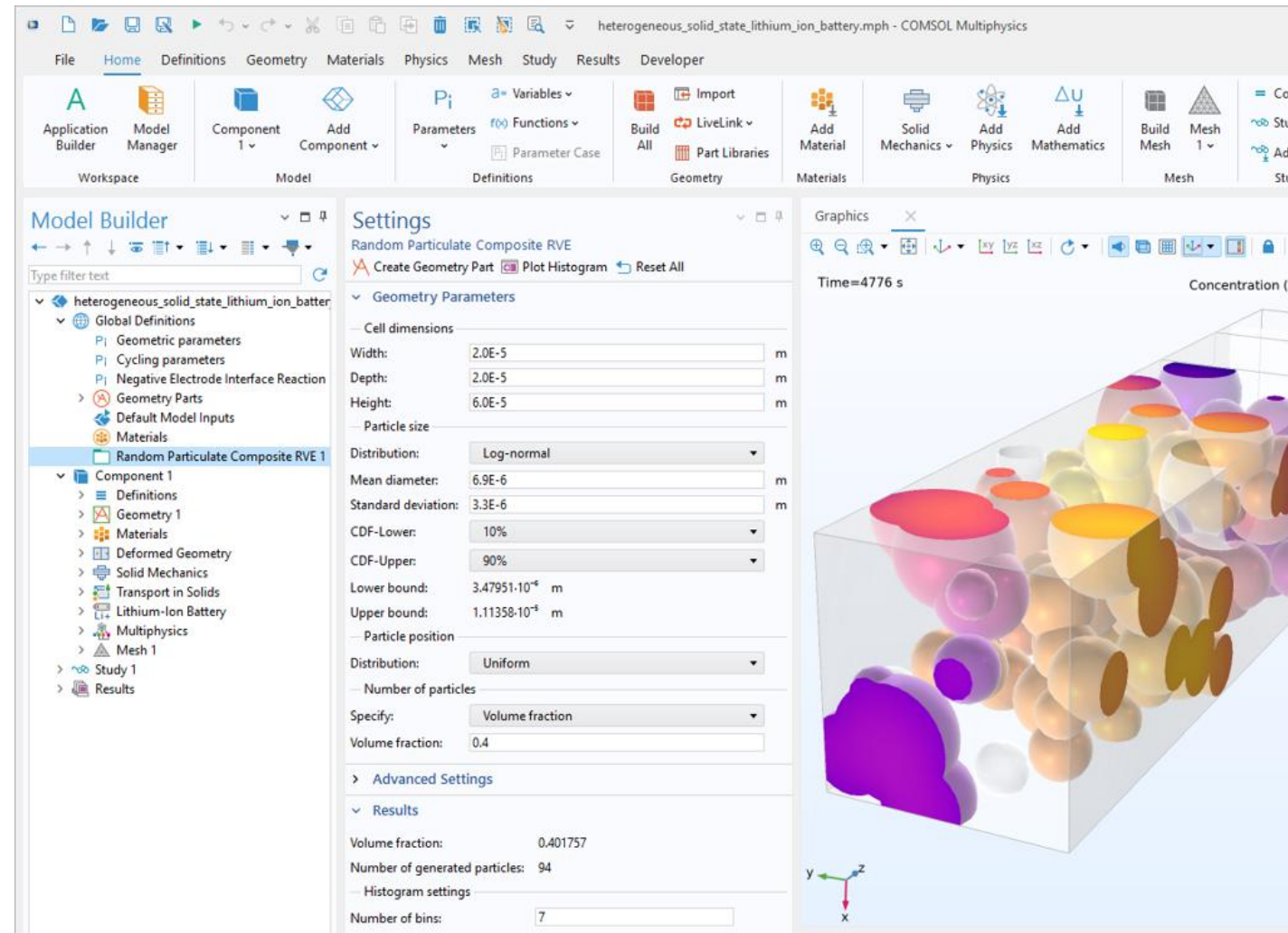
# Analytical Approach

- Closed-form formulas — no simulation needed
- For fast estimate
- Built-in mixing rules in Effective Material
  - Volume/mass averages, etc.
- Limited for complex microstructures and anisotropy



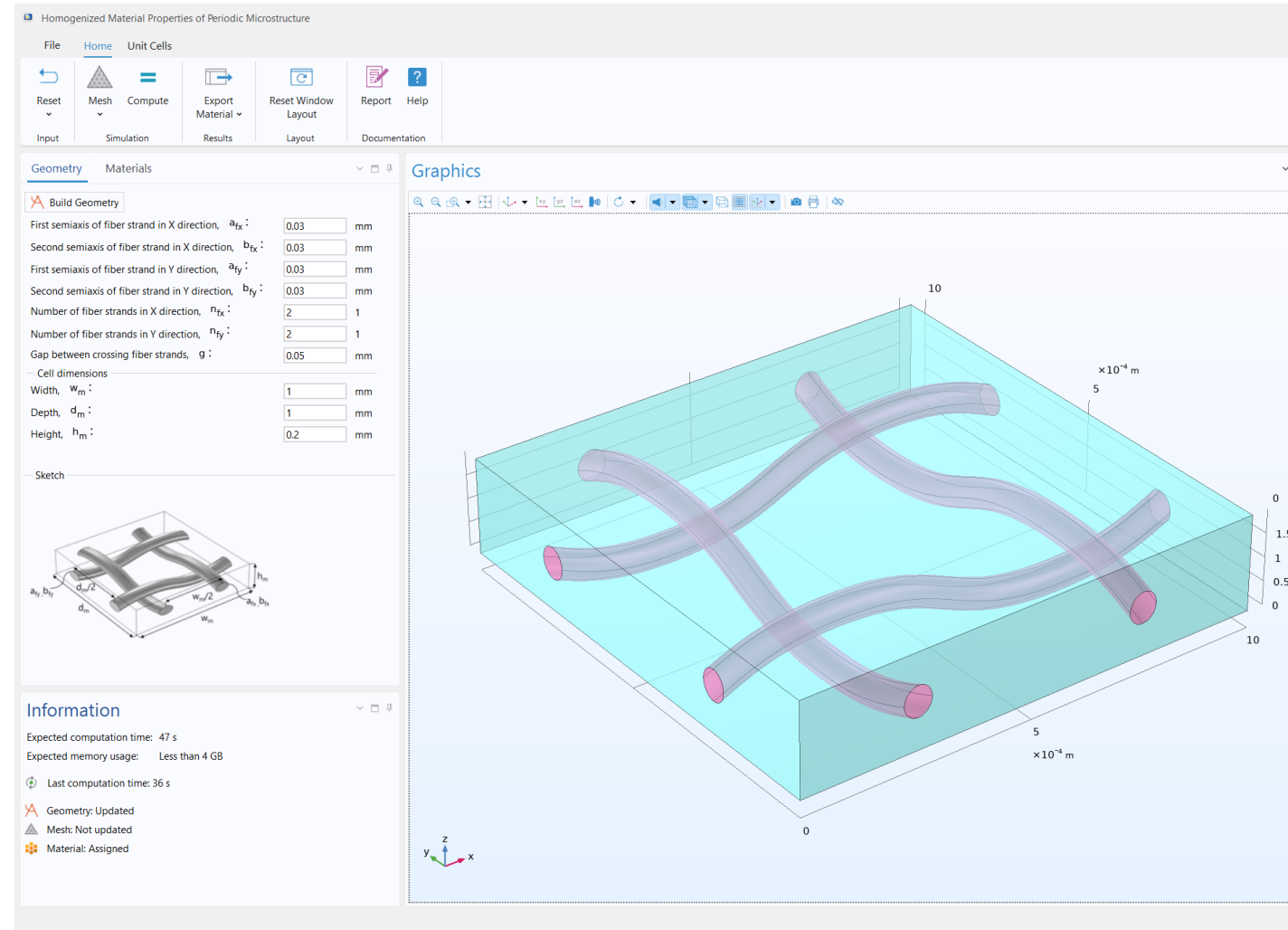
# Random Particulate Composite RVE

- COMSOL Multiphysics Add-in
- Builds RVEs with random spherical particles
- Specify either number of particles or volume fraction
- Overlap control



# Homogenization App

- Homogenized structural and thermal properties
- Ten different unit cell geometries
- Flexibility to modify the geometry and material properties
- Homogenized material can be exported



## Try It Yourself

- Try it on our COMSOL Server
- Link: [vulpecula.humusoft.cz:2036](http://vulpecula.humusoft.cz:2036)
- Username / Password: showcase / showcase
  
- Questions?

