A DSGE Model of the Czech Inflation Targeting and Monetary Policy:

Solving and Estimating

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- **Linearized System**
- **Solving the Model**
- **Estimation of the Model**
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The Linearized System

Transformation of the variables:

$y_t = Y_t/Z_t$, $c_t = C_t/Z_t$, $\lambda_t = Z_t\Lambda_t$, $z_t = Z_t/Z_{t-1}$ (with a reference to Z_t) and $\pi_t = \Pi_t/\Pi_t^*$, $r_t = R_t/\Pi_t^*$ and $\pi_t^* = \Pi_t^*/\Pi_{t-1}^*$ (with a reference to Π_t^*).

Conditions that hold in steady state:

$a = 1, \pi^* = 1, v = 1, \pi = 1, g^\pi = 1, g^r = 1, a_t = \theta, z_t = z$ and after next calculations we get: $g^y = z, \lambda = \theta/(\theta - 1), y = \{(\theta-1)/\theta\}\{(z-\beta\gamma)/(z-\gamma)\}$, $r = z/\beta$ and $r^{r\pi} = z/\beta$ for t = 1, 2, 3, ...

Percentage deviation of the variable from steady state value:

$\hat{y}_t = ln(y_t/y), \hat{c}_t = ln(c_t/c), \hat{\pi}_t = ln(\pi_t), \hat{r}_t = ln(r_t/r), \hat{g}_t^y = ln(g_t^y/g^y), \hat{g}_t^\pi = ln(g_t^\pi), \hat{y}_t = ln(y_t/y), \hat{g}_t^r = ln(g_t^r), \hat{r}_t^{r\pi} = ln(r_t^{r\pi}/r^{r\pi}), \hat{\lambda}_t = ln(\lambda_t/\lambda), \hat{a}_t = ln(a_t), \hat{\theta}_t = ln(\theta_t/\theta), \hat{z}_t = ln(z_t/z), \hat{v}_t = ln(v_t)$, and $\hat{\pi}_t^* = ln(\pi_t^*)$.

The system of the stationary equations log-linearized around the steady state. A first–order approximation of equations imply (for all t = 1, 2, ...)

$$(z-\gamma)(z-\beta\gamma) = \gamma z\hat{y}_{t-1} + \beta\gamma z E_t\hat{y}_{t+1} - (z^2+\beta\gamma^2)\hat{y}_t + (z-\gamma)(z-\beta\gamma\rho_a)\hat{a}_t - \gamma z\hat{z}_t \quad (1)$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{r}_t - E_t\hat{\pi}_{t+1} \quad (2)$$

$$(1+\beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t+1} + \beta E_t\hat{\pi}_{t+1} + \psi(\hat{a}_t - \hat{\lambda}_t) - \hat{e}_t - \alpha\hat{\pi}_t^* \quad (3)$$

$$\hat{r}_t - \hat{r}_{t-1} = \rho_\pi\hat{\pi}_t + \rho_{gy}\hat{g}_t^y - \pi_t^* + \hat{v}_t \quad (4)$$

$$\hat{\pi}_t^* = \sigma_\pi\epsilon_{\pi t} - \delta_e\epsilon_{et} - \delta_z\epsilon_{zt} \quad (5)$$

$$\hat{g}_t^y = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \quad (6)$$

$$\hat{g}_t^\pi = \hat{\pi}_t - \hat{\pi}_{t-1} + \hat{\pi}_t^* \quad (7)$$

$$\hat{r}_t^{r\pi} = \hat{r}_t - \hat{\pi}_t \quad (8)$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \sigma_a\epsilon_{at} \quad (9)$$

$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \sigma_e\epsilon_{et} \quad (10)$$

$$\hat{z}_t = \sigma_z\epsilon_{zt} \quad (11)$$

$$\hat{v}_t = \rho_v\hat{v}_{t-1} + \sigma_v\epsilon_{vt} \quad (12)$$

The new variables are:

$\hat{e}_t = (1/\phi)\hat{\theta}_t, \psi = (\theta - 1)/\phi, \delta_e = \delta_\theta$ and $\sigma_e = \sigma_\theta/\phi$.

The first five equations are the ground of the model:

- Marginal utility of households' consumption (1)
- New Keynesian IS curve (2)
- New Keynesian Phillips curve (3)
- Monetary policy (4) – (5)

The subsequent three equations (6) – (8) state the definitions of the growth rate for the output, the inflation rate and the nominal interest rate to the inflation.

The rest equation describes

- shock to the households' preference (9)
- cost–push shock (10)
- technology shock (11)
- monetary shock (12).

There are 17 parameters:

$z, \beta, \psi, \gamma, \alpha, \rho_\pi, \rho_{gy}, \rho_a, \rho_e, \rho_v, \sigma_a, \sigma_e, \sigma_z, \sigma_v, \sigma_\pi, \delta_e$ and δ_z

and 3 observable variables

- the growth rate of the output ($g_t^y = y_t/y_{t-1}$),
- the growth rate of the inflation ($g_t^\pi = \pi_t/\pi_{t-1}$) and
- the growth rate of the nominal interest rate to the inflation ($r_t^{r\pi} = r_t/\pi_t$).

Solving the Model

Let

$$s_t^0 = \left[\hat{y}_{t-1} \;\; \hat{\pi}_{t-1} \;\; \hat{r}_{t-1} \;\; \hat{q}_{t-1} \;\; \hat{\lambda}_t \;\; \hat{y}_t \;\; \hat{\pi}_t \;\; \hat{q}_t \right]^T$$

and

$$\xi_t = \left[\hat{a}_t \;\; \hat{e}_t \;\; \hat{z}_t \;\; \hat{v}_t \;\; \hat{\pi}_t^* \right]^T$$

Then (1) , (2), (3) and (4)

$$AE_t \, s_{t+1}^0 = Bs_t^0 + C\xi_t \qquad (13)$$

Meanwhile (5), (9), (10), (11) and (12)

$$\xi_t = P\xi_{t-1} + X\varepsilon_t \qquad (14)$$

where

$$\varepsilon_t = \left[\varepsilon_{at} \;\; \varepsilon_{et} \;\; \varepsilon_{zt} \;\; \varepsilon_{vt} \;\; \varepsilon_{\pi t} \right]^T$$

Equation (13) takes the form of *linear expectational difference equations*, driven by the *exogenous shocks* in (14).
The system can be solved by uncoupling *the ustable* and *stable components* and than solving unstable component forward.
The approach taken here uses the methods outlined by Klein.

Klein`s method relies on *the complex generalized Schur decomposition,* which identifies unitary matrices *Q* and *Z* such than

$$QAZ = S$$

and

$$QBZ = T$$

are both upper triangular, where the generalized eigenvalues of *B* and *A* can be recovered as the ratios the diagonal elements of *T* and *S*

$$\lambda(B, A) = \{t_{ii} / s_{ii} \mid i = 1, 2, ..., 8\}$$

There are four predetermited variables in the vector s_t^0 .Thus, if *four* generalized eigenvalues in $\lambda(B, A)$ lie *inside* the unit circle and *four* of the generalized eigenvalues lie outside the unit circle, then the system hes *a unuque solution*.

If *more then four* generalized eigenvalues in $\lambda(B, A)$ lie *outside* the unit circle, than the system has *no solution.*

If *less than four* generalized eigenvalues in $\lambda(B, A)$ lie outside the unit circle, then solution has *multiple solutions*.

See Blanchard and Kahn (1980) and Klein (2000).

Assumed from now on that there exactly four generalized eigenvalues that lie outside the unit circle, and partion the matrices *Q, Z, S,* and *T*

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

where Q_1 and Q_2 are both 4 x 8, and

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \qquad S = \begin{bmatrix} S_{11} & S_{12} \\ 0_{(4x4)} & S_{22} \end{bmatrix} \qquad T = \begin{bmatrix} T_{11} & T_{12} \\ 0_{(4x4)} & T_{22} \end{bmatrix}$$

The solution to be writen more conveniently as

$$\begin{bmatrix} \hat{\lambda}_{\mathrm{t}} \\ \hat{y}_{\mathrm{t}} \\ \hat{\pi}_{\mathrm{t}} \\ \hat{q}_{\mathrm{t}} \end{bmatrix} = M_1 \begin{bmatrix} \hat{y}_{\mathrm{t-1}} \\ \hat{\pi}_{\mathrm{t-1}} \\ \hat{r}_{\mathrm{t-1}} \\ \hat{q}_{\mathrm{t-1}} \end{bmatrix} + M_2 \xi_t \qquad (15)$$

$$M_1 = Z_{21} Z_{11}^{-1}$$

$$M_2 = -\left[Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}\right] T_{22}^{-1} R$$

Substitute results to obtain the solution

$$\begin{bmatrix} \hat{\lambda}_{\mathrm{t}} \\ \hat{y}_{\mathrm{t}} \\ \hat{\pi}_{\mathrm{t}} \\ \hat{q}_{\mathrm{t}} \end{bmatrix} = M_3 \begin{bmatrix} \hat{y}_{\mathrm{t-1}} \\ \hat{\pi}_{\mathrm{t-1}} \\ \hat{r}_{\mathrm{t-1}} \\ \hat{q}_{\mathrm{t-1}} \end{bmatrix} + M_4 \xi_t \qquad (16)$$

$$M_3 = Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}$$

$$M_4 = Z_{11} S_{11}^{-1} \left(T_{11} Z_{11}^{-1} Z_{12} T_{22}^{-1} R + Q_1 C + S_{12} T_{22}^{-1} RP - T_{12} T_{22}^{-1} R\right) - Z_{12} T_{22}^{-1} RP$$

To complete the construction of the solution, let

$$s_t = \left[\hat{y}_{\mathrm{t-1}} \;\; \hat{\pi}_{\mathrm{t-1}} \;\; \hat{r}_{\mathrm{t-1}} \;\; \hat{q}_{\mathrm{t-1}} \;\; \hat{a}_{\mathrm{t}} \;\; \hat{e}_{\mathrm{t}} \;\; \hat{z}_{\mathrm{t}} \;\; \hat{v}_{\mathrm{t}} \;\; \hat{\pi}_t^{*}\right]^T$$

$$\Pi = \begin{bmatrix} M_3 & M_4 \\ 0_{(5x4)} & P \end{bmatrix} \qquad W = \begin{bmatrix} 0_{(5x4)} \\ X \end{bmatrix}$$

The compactly solution

$$s_{t+1} = \Pi s_t + W\varepsilon_{t+1} \tag{17}$$

Estimation of the Model

The empirical model hes 19 parameters

z, β, γ, α ,ψ, ς_{π}, ς_{x}, ς_{gy}, ς_{a}, ς_{e}, ς_{v}, σ_{a}, σ_{e}, σ_{z}, σ_{v}, σ_{π}, δ_{a}, δ_{e} and δ_{z}

Data:

$$d_{\mathrm{t}} = \begin{bmatrix} \hat{g}_t^{y} \\ \hat{g}_t^{\pi} \\ \hat{g}_t^{r\pi} \end{bmatrix}$$

The empirical model for estimating

$$s_{t+1} = As_t + B\varepsilon_{t+1} \tag{18}$$

$$d_t = Cs_t \tag{19}$$

where

$$A = \Pi, \quad B = W$$

Note (19) that

$$\hat{d}_{t|t-1} = C\hat{s}_{t|t-1}$$

Hence

$$u_t = d_t - \hat{d}_{t|t-1} = C\left(s_t - s_{t|t-1}\right)$$

in such that

$$Eu_t u_t^{\mathrm{T}} = C\Sigma_{t|t-1}C_t^{\mathrm{T}}$$

Hamilton`s formula for updating a linear projection

$$\hat{s}_{t|t} = \hat{s}_{t|t-1} + \Sigma_{t|t-1}C_t^{\mathrm{T}}\left(C\Sigma_{t|t-1}C_t^{\mathrm{T}}\right)^{-1}u_t$$

Hence from (18)

$$\hat{s}_{t+1|t} = A\hat{s}_{t|t-1} + A\Sigma_{t|t-1}C_t^{\mathrm{T}}\left(C\Sigma_{t|t-1}C_t^{\mathrm{T}}\right)^{-1}u_t .$$

Using this last result, along with (18)

$$s_{t+1} - \hat{s}_{t+1|t} = A\hat{s}_{t|t-1} + A\left(s_t - s_{t|t-1}\right) + B\varepsilon_t - A\Sigma_{t|t-1}C^{\mathrm{T}}\left(C\Sigma_{t|t-1}C^{\mathrm{T}}\right)^{-1}u_t$$

Hence

$$\Sigma_{t+1|t} = BVB^{\mathrm{T}} + A\Sigma_{t|t-1}A^{\mathrm{T}} - A\Sigma_{t|t-1}C^{\mathrm{T}}\left(C\Sigma_{t|t-1}C^{\mathrm{T}}\right)^{-1}C\Sigma_{t|t-1}A^{\mathrm{T}}$$

The results can be summarized as follows.

$$s_t = \hat{s}_{t|t-1} = E\left(s_t \mid d_{t-1}, d_{t-2}, ..., d_1\right)$$

and

$$\Sigma_t = \Sigma_{t|t-1} = E\left(s_t \mid d_{t-1}, d_{t-2}, ..., d_1\right)_{t|t-1} = E\left(s_t - \hat{s}_{t|t-1}\right)\left(s_t - \hat{s}_{t|t-1}\right)^{\mathrm{T}}$$

Than

$$\hat{s}_{t+1} = A\hat{s}_t + K_t u_t$$

and

$$d_t = C\hat{s}_t + u_t$$

where

$$u_t = d_t + E(d_t \mid d_{t-1}, d_{t-2}, ..., d_1)$$

$$Eu_t u_t^{\mathrm{T}} = C\Sigma_{t|t-1}C_t^{\mathrm{T}} = \Omega_t$$

the sequences for K_t a Σ_t can be generated recursively

$$K_t = A\Sigma_t C^{\mathrm{T}} \left(C\Sigma_{t|t-1}C^{\mathrm{T}}\right)^{-1}$$

and

$$\Sigma_t = BVB^{\mathrm{T}} + A\Sigma_{t|t-1}A^{\mathrm{T}} - A\Sigma_{t|t-1}C^{\mathrm{T}} \left(C\Sigma_{t|t-1}C_t^{\mathrm{T}}\right)^{-1} C\Sigma_{t|t-1}A^{\mathrm{T}},$$

and initial conditions

$$\hat{s}_{1|0} = Es_1 = 0_{(9x1)},$$

$$vec(\Sigma_{1|0}) = vec(Es_1 s_{1\,t}^{\mathrm{T}}).$$

The inovations $\{u_t, t = 1,2,...,T\}$ than can be used to form log likelihood function for $\{d_t, t = 1,2,...,T\}$ as

$$\ln L = -(3\mathrm{T}/2\pi)\ln(2\pi) - 1/2\ \Sigma \ln|\Omega_t| - 1/2\ \Sigma\, u_t \Omega_t^{-1} u_t^{\mathrm{T}}$$

Results of Estimation

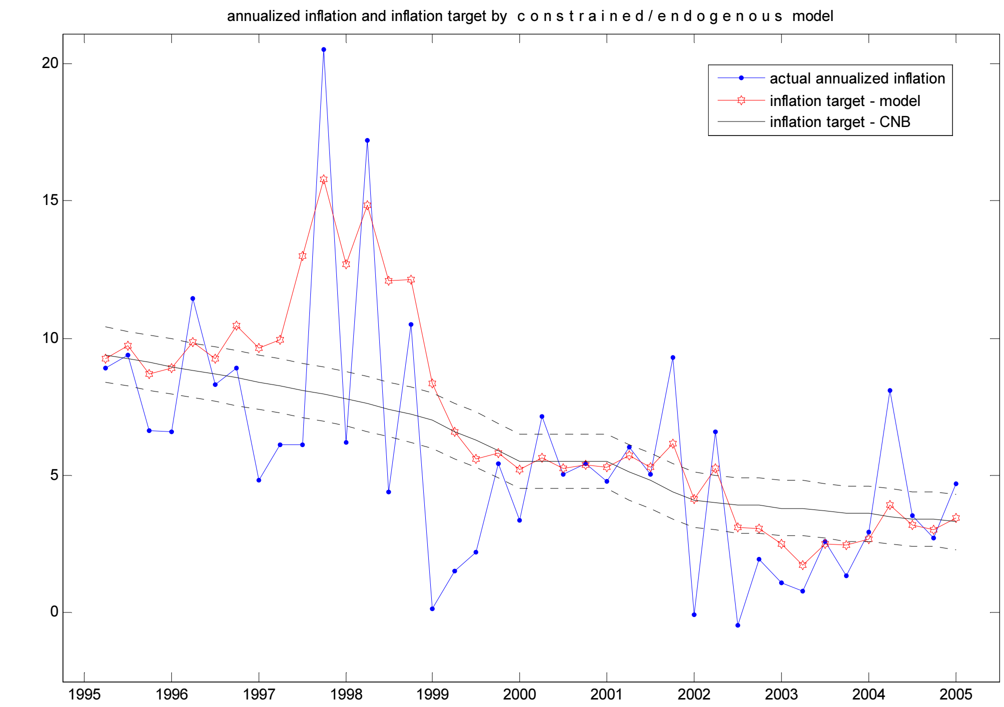
Estimates of the Model with Endogenous Target

Parameter	Estimate	Standard Deviations Target
z	1.0057	0
β	0.99778	0
γ	0.91272	0.025536
α	0	*
ψ	0.1	0
ς_{π}	0.30946	0.080288
ς_{gy}	0.023574	0.069327
ς_{a}	0	*
ς_{e}	0	*
ς_{v}	0	*
σ_{a}	0.0240390	0.0062766
σ_{e}	0.0041403	0.0018223
σ_{z}	0.0311820	0.0052965
σ_{v}	0.0049429	0.0006269
σ_{π}	0.0024665	**
δ_{e}	0.0023736	0.0006249
δ_{z}	0.0020351	0.0009140

J = 456.871

* the estimate lies up against the boundary of the parameter space,
** the calibrated value
J the maximum value of log likelihood function

Cunsumer price inflation and inflation target (annualized)



Impulse responses (percentage-point) to a one standart deviation shocks