ADVANTAGES OF TYPE-2 FUZZY LOGIC IN NETWORKED CONTROL SYSTEMS

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Abstract

The popularity of Networked Control Systems has increased in recent years. Data transfers between two points of the network induce network delays and these delays are variable. Uncertainties such as variable time delays and packet dropouts must be covered by the control strategy design of the Networked Control Systems. In fuzzy type-2 sets the uncertainty is represented as an extra dimension. In this article we show how it is possible to reduce the effect of network induced variable time delays in Networked Control Systems with the framework of type-1 and type-2 fuzzy logic.

1 Introduction

The theory of control and its applications has evolved for many years. Trends from other domains arrives into the control theory with beneficial advantages. The goal is to implement this trends to the control systems. Implementation brings not only advantages but some difficulties too. We must be therefore careful to use these trends without loosing the robustness and stability of the control system. One of these modern trends are data networks in control loop. Data networks enable remote data exchange among users, they reduce network wiring, network complexity, maintenance and costs. The uncertainties that affect the control loop are variable time delays and packet dropout [1].

As we mentioned robustness and stability, the uncertainties can badly influence these properties of control systems. One of the most known control strategies that deals with uncertainties is Fuzzy Control. Last decade scientists explored how uncertainties affect FLS. They have shown that standard fuzzy logic can't handle uncertainties because (i) the words mean different things to different people, the meanings of the words that are used in antecedents and consequents of rules can be uncertain, (ii) consequents may have a histogram of values associated with them, especially when knowledge is extracted from a group of experts who do not all agree, (iii) measurements that activate a standard FLS may be noisy and therefore uncertain, and (iv) finally, the data that are used to tune the parameters of a standard FLS may also be noisy. Mendel and other scientists developed enhanced standard (type-1) FLS which are nowadays referred as type-2 FLS [2,3].

This paper is divided into following sections. In second section we describe the Networked Control Systems (NCS) and their properties. We show how variable time delays affect the controller performance. In third section we describe most common Fuzzy Logic Systems (FLS) which are using type-1 fuzzy sets. Fourth section describes FLS with type-2 fuzzy sets and advantages which can be gained. Last section summarizes all results.

2 Networked Control Systems

Nowadays we can see the trend of passing from the traditional centralized control to distributed control systems. This calls for a change of the usual design approaches to the control systems. The traditional automation is fusing with the technologies known from informatics and computer networks [4]. According to [5], a Networked Control System (NCS) is a distributed control structure where the communication between the nodes of the control system is provided by a communication network. The basic elements of a NCS are sensors, controllers, actuators and the communication network.

According to the communication point of view, the NCSs can be divided into straight and hierarchical [5]. In a straight NCS, the control loop consists of a controller, sensors and actuators connected through a network. In the hierarchical NCS the system consists of local straight control loops connected to a superior controller. This type of NCS is used in more complicated systems, like in mobile robotics. Following the successful implementation of wired Ethernet technologies in industry the manufacturers have been developed also the industrial implementation of the wireless data transfer technologies.

Some difficulties come with new possibilities and advantages of NCS. Data transfers between two points of the network induce network delays and these delays are variable. We must consider three common delays in the control loop (Figure 1.). The first delay is between the controller and the actuator, the second between the sensor and the controller. The third delay is the computational time of the controller. The computational delay can be incorporated into the two delays mentioned before. Another problem of NCS is the packet dropout.

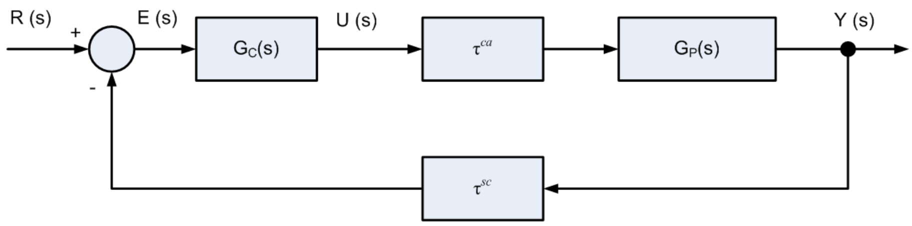

Figure 1: Networked Control System

Delays in a control loop degrade the system performance of a control system, so do the network delays in a NCS [1]. In [1] it is showed the system performance of the PI controller with delays in the loop. The transfer function of the controller was following

$$G_C(s)=\frac{\beta K_P\left(s+\left(K_P/K_I\right)\right)}{s} \quad \beta=1,\ \ K_P=0.1702,\ \ K_I=0.378 \tag{1}$$

and the transfer function of the plant was

$$G_P(s)=\frac{2029.826}{(s+26.29)(s+2.296)} \tag{2}$$

When the delays are longer, the system overshoot is higher and the settling time is longer. On the figure 2 we show these effects

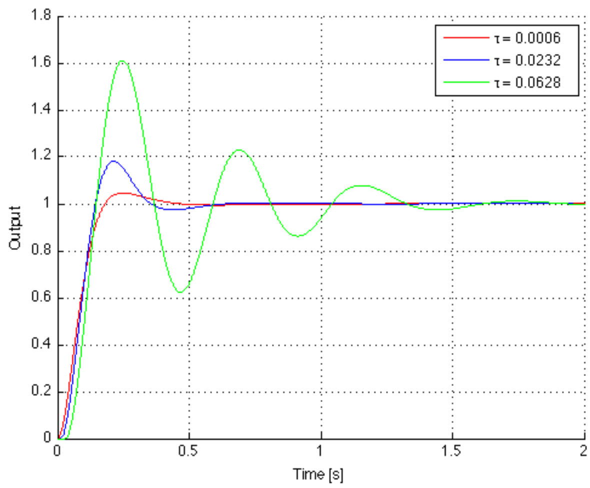

Figure 2: System performance with delays in the loop

3 Type-1 fuzzy logic system

Fuzzy logic has evolved for more than 40 years. The type-1 FLS is the most know and widely used FLS. It has been successfully implemented in many real world applications. It contains the fuzzifier, the inference mechanism, the rules and the deffuzifier as you can see on figure 3.

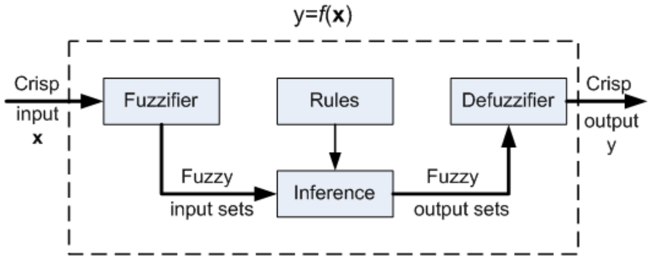

Figure 3: Type-1 fuzzy logic system

The rules represent the relation between input and output space. The most traditional rules has multiple inputs and one output. For p inputs an one output we can write lth rule as [2]

$$R^l : IF\ x_1\ is\ X_1 \text{ and} \ldots \text{and } x_p\ is X_p\ THEN\ y \text{ is } G^l \quad l=1,\ldots,M \tag{3}$$

where M is the number of rules. The $X_{1,} \ldots, X_p \text{ and } G$ are type-1 fuzzy membership functions

$$A=\{(x, \mu_A(x)) | \forall x \in X\} \tag{4}$$

where $\mu_A(x)$ are constraints between 0 a 1 for $x \in X$. Fuzzifier maps crisps inputs into fuzzy sets and inference mechanism combine rules. We described this two parts in the next subsection because they change with the fuzzifier type. To obtain crisp input from fuzzy sets FLS uses a deffuzifier. In this paper we use the well known Gaussian membership functions, the product t-norm, the maximum t-conorm as the union and the height deffuzifier for its simplicity.

3.1 Singleton Type-1 FLS

Singleton type-1 FLS are the most common and most widely used. Singleton type-1 FLS use fuzzy singletons in the fuzzification process. As we mentioned before, the fuzzifier maps crisps inputs into fuzzy sets. A fuzzy singleton has $\mu_{X_i}(x_i')=1\ (i=1,\ldots,p)$ when $x=x_i'$ and $\mu_{X_i}(x_i')=0\ (i=1,\ldots,p)$ for all others. In [2] inference mechanism for type-1 FLS has the following form

$$\mu_{B^l}(y)=\mu_{G^l}(y) \star \left\{\left[\sup_{x_1 \in X_i} \mu_{X_1}(x_1) \star \mu_{F_1^l}(x_1)\right] \star \ldots \star \left[\sup_{x_1 \in X_i} \mu_{X_p}(x_p) \star \mu_{F_p^l}(x_p)\right]\right\} \tag{5}$$

where the star symbol means in our case t-norm (product). At point $x_i=x_i'$ we can (5) simplify because of the singleton fuzzification into the form

$$\mu_{B^l}(y)=\mu_{G^l}(y) \star \left[\mu_{X_1}(x_1') \star \ldots \star \mu_{X_p}(x_p')\right] \tag{6}$$

In our simulation for singleton type-1 FLS we used Gaussian membership functions, max-prod inference mechanism and height deffuzification. The rules were computed based on PI controller from equation (1). We uniformly covered the universe from inputs (in our case error and integral of error) and the corresponding output was computed by multiplying the mean and standard deviation by P and I component of PI controller. Now we had an equivalent fuzzy PI controller which responses were similar to the ones on the figure 2.

3.2 Non-singleton Type-1 FLS

A non-singleton type-1 FLS is a type-1 FLS whose inputs are modeled as type-1 fuzzy numbers. This can be used to handle uncertainties that occur at the input, for example noisy measurements. A non-singleton type-1 FLS is described by the same figure 3 as the singleton FLS. The difference is the fuzzifier which treats the inputs as type-1 fuzzy sets and the effect of this on inference block. A non-singleton fuzzifier is one for which $\mu_{X_i}(x_i')=1\ (i=1,..p)$ and $\mu_{X_i}(x_i)$ decreases form unity as x_i moves away from x_i'. This means that the measured value is most likely the correct value, but because the noise neighboring values are also likely to be the correct values, but to a lesser degree. [2] The shape of the membership function of our input was modeled as a Gaussian membership function.

Non-singleton type-1 FLS first pre-filters its inputs **x**, transforming it to $x^l_{k,\max}$. This incorporates an input uncertainty into FLS. Pre-filtering of input is depicted in figure 4.

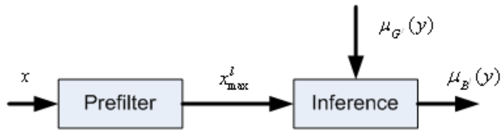

Figure 4: Pre-filtering of the input

Mouzouris and Mendel [6] showed how to compute $x^l_{k,\max}$. Because the time delays in NCS are variable the non-singleton fuzzification could improve closed loop performance. As you can see on the figure 5 the overshoot of the non-singleton fuzzification is smaller.

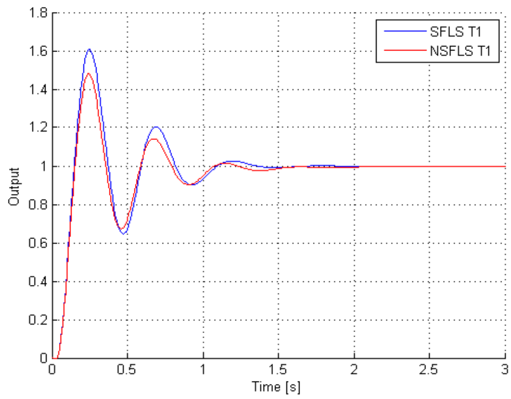

Figure 5: Closed loop system with singleton type-1 and non-singleton type-1 FLS

4 Type-2 fuzzy logic system

The last decade started a new direction in FLS. Scientists "led" by Mendel "dusted off" the ideas of Zadeh that he had on enhanced fuzzy sets (type-2 fuzzy sets) [2]. In fuzzy type-2 sets the uncertainty is represented as an extra dimension. A type-2 fuzzy set $\tilde{A}$ is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{7}$$

where $0 \le \mu_{\tilde{A}}(x,u) \le 1$. A great computational simplification is when we put $\mu_{\tilde{A}}(x,u) = 1$. In this case we talk about interval type-2 fuzzy sets which we use heavily in this article. Example of interval type-2 fuzzy sets are Gaussian membership functions with uncertain mean or uncertain standard deviation defined as (figure 6):

$$\mu_A(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right] \quad m \in [m_1, m_2] \text{ or } \sigma = [\sigma_1, \sigma_2] \tag{8}$$

New in type-2 fuzzy sets is the union (join $\sqcup$), the intersection (meet $\sqcap$) and the complement (negation $\neg$). Unfortunately their description exceeds this article, but the readers can find it in literature for type-2 fuzzy sets. The type-2 FLS has the same schema as type-1 FLS with difference in output processing as you can see on figure 7. Rules of type-2 FLS contain type-2 fuzzy sets (at least one)

$$R^l : IF\ x_1\ is\ \tilde{X}_1 \text{ and} \ldots \text{and } x_p\ is\ \tilde{X}_p\ THEN\ y \text{ is } \tilde{G}^l \quad l = 1, \ldots, M \tag{9}$$

where M is the number of rules. The $\tilde{X}_1, \dots, \tilde{X}_p$ and $\tilde{G}$ are type-2 fuzzy membership functions. Type reduction reduces the type-2 fuzzy sets into type-1 fuzzy sets and can be made by iterative algorithm developed by Karnik and Mendel [7].

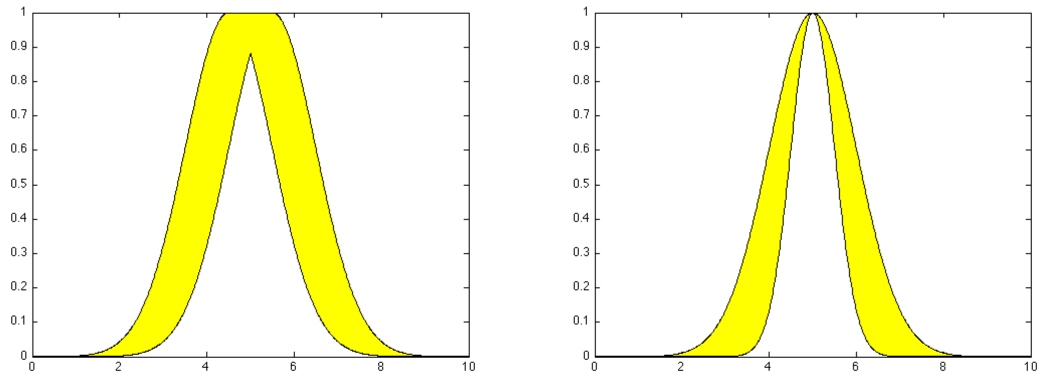

Figure 6: Gaussian primary MF with uncertain mean (left) and standard deviation (right)

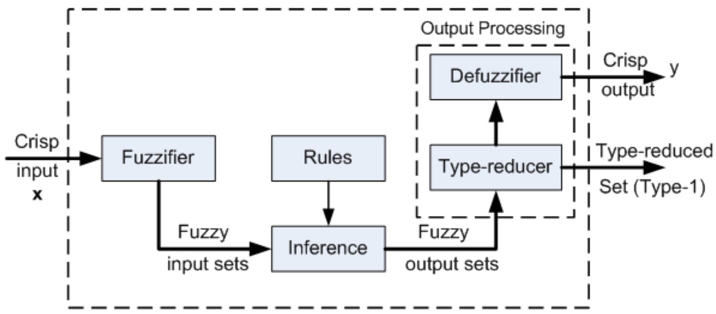

Figure 7: Type-2 fuzzy logic system

4.1 Singleton Type-2 FLS

As we mentioned before singleton type-2 FLS use type-2 fuzzy sets in the inference mechanism. The general representation of singleton type-2 FLS inference mechanism can be described by following equations (Mendel [2]).

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap\left\{\left[\sqcup_{x_1\in X_1}\mu_{\tilde{X}_1}(x_1)\sqcap\mu_{\tilde{F}_1^l}(x_1)\right]\sqcap\dots\sqcap\left[\sqcup_{x_p\in X_p}\mu_{\tilde{X}_p}(x_p)\sqcap\mu_{\tilde{F}_p^l}(x_p)\right]\right\} \quad (10)$$

The fuzzification is similar to non-singleton type-1 FLS. It uses type-2 fuzzy singletons so the equation (10) can be simplified to this form at $x_i=x_i'$

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap\left\{\sqcap_{i=1}^{p}\mu_{\tilde{F}_i^l}(x_i')\right\} \quad (11)$$

In our simulation we changed type-1 fuzzy sets to type-2 fuzzy sets. We used interval type-2 Gaussian membership functions with uncertain mean. The closed loop response of the system is very good. As you can see the overshoot of the system is smaller and the response isn't much damped.

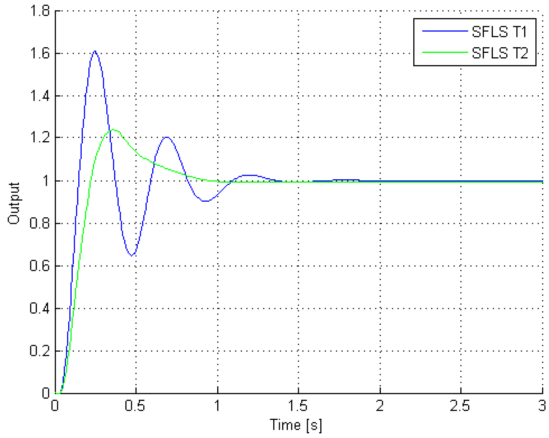

Figure 8: Closed loop system with singleton type-1 and singleton type-2 FLS

4.2 Type-1 Non-Singleton Type-2 FLS

The inputs of the type-1 non-singleton type-2 FLS are modeled as type-1 fuzzy numbers. This is beneficial when inputs are noisy or varying like in NCS. The type-1 non-singleton type-2 FLS are like type-2 FLS with difference in the fuzzification and it's effect on the inference. Mendel [2] described the inference of type-1 non-singleton type-2 FLS with the following equations

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap\left\{\left[\sqcup_{x_1\in X_1}\mu_{X_1}(x_1)\sqcap\mu_{\tilde{F}_1^l}(x_1)\right]\sqcap\ldots\sqcap\left[\sqcup_{x_p\in X_p}\mu_{X_p}(x_p)\sqcap\mu_{\tilde{F}_p^l}(x_p)\right]\right\} \quad (12)$$

where l is the number of rules. Let

$$\mu_{\tilde{Q}_k^l}(x_K)\equiv\mu_{X_k}(x_k)\sqcap\mu_{\tilde{F}_k^l}(x_k) \quad (13)$$

where l is the number of rules and k is the number of inputs. Let

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap\left\{\sqcup_{x\in X}\left[\sqcap_{k=1}^{p}\mu_{\tilde{Q}_k^l}(x_k)\right]\right\} \quad (14)$$

then $\mu_{\tilde{B}^l}(y)$ can be re-expressed as

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap F^l(x') \quad (15)$$

The term $F^l(x')$ is the firing set for a type-1 non-singleton type-2 FLS. In our simulations we also used a simplification from general type-2 fuzzy membership functions to interval type-2 fuzzy membership functions. The inference for interval type-1 non-singleton type-2 FLS that we used in this paper is described in [2] under theorem 11-1. For input measurements we used Gaussian membership functions. The comparison between type-1 singleton FLS and type-1 non-singleton type-2 FLS you can see in figure 9 . As you can see type-1 non-singleton type-2 FLS can handle uncertainties in NCS much better.

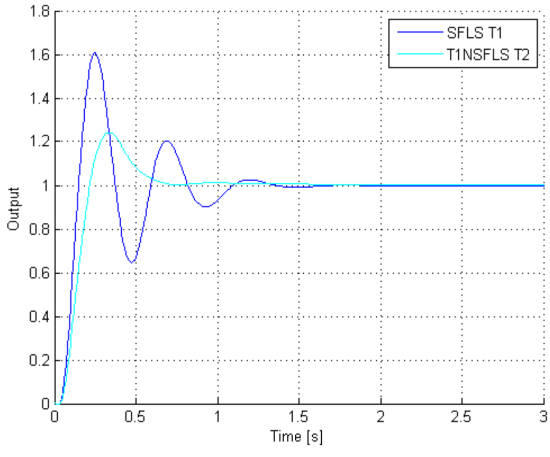

Figure 9: Closed loop system with singleton type-1 and type-1 non-singleton type-2 FLS

4.3 Type-2 Non-Singleton Type-2 FLS

In type-2 non-singleton type-2 FLS inputs are modeled as type-2 fuzzy numbers. This can be useful in NCS because of the variable time delays in the loop. Even more because the data networks can provide different quality of service over time and this changes the characteristic of variable time delays in the loop. Mendel [2] described the inference of type-2 non-singleton type-2 FLS with following equations

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap\left\{\left[\sqcup_{x_1\in X_1}\mu_{\tilde{X}_1}(x_1)\sqcap\mu_{\tilde{F}_1^l}(x_1)\right]\sqcap\ldots\sqcap\left[\sqcup_{x_p\in X_p}\mu_{\tilde{X}_p}(x_p)\sqcap\mu_{\tilde{F}_p^l}(x_p)\right]\right\} \qquad (16)$$

where l is the number of rules. Let

$$\mu_{\tilde{Q}_k^l}(x_K)\equiv\mu_{\tilde{X}_k}(x_k)\sqcap\mu_{\tilde{F}_k^l}(x_k) \qquad (17)$$

where l is the number of rules and k is the number of inputs. Let

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap\left\{\sqcup_{x\in X}\left[\sqcap_{k=1}^{p}\mu_{\tilde{Q}_k^l}(x_k)\right]\right\} \qquad (18)$$

than $\mu_{\tilde{B}^l}(y)$ can be re-expressed as

$$\mu_{\tilde{B}^l}(y)=\mu_{\tilde{G}^l}(y)\sqcap F^l(x') \qquad (19)$$

The term $F^l(x')$ is the firing set for a type-2 non-singleton type-2 FLS. It contains the effect of input uncertainties and uncertainties in antecedent. Again we used a simplification from general type-2 fuzzy membership functions to interval type-2 fuzzy membership functions. The inference for interval type-2 non-singleton type-2 FLS that we used in this paper is described in [2] under theorem 12-1. For input measurements we used Gaussian membership functions with uncertain deviation. The comparison between type-1 singleton FLS and type-2 non-singleton type-2 FLS you can see in figure 10. As you can see type-2 non-singleton type-2 FLS again handled uncertainties in NCS much better.

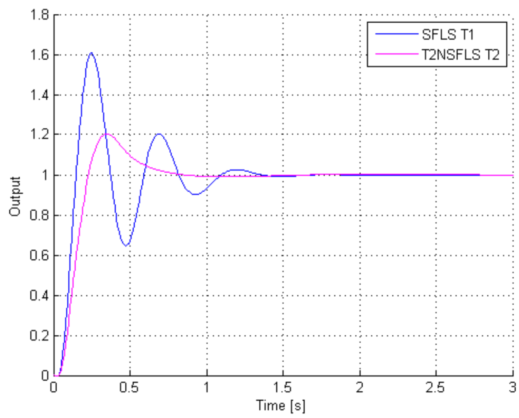

Figure 10: Closed loop system with singleton type-1 and type-2 non-singleton type-2 FLS

5 Results

Finally we compared all FLS mentioned in this paper. The pre-computed type-1 FLS and PI controller were almost identical. The advantage of the fuzzy controller was that we could incorporate the uncertainties into different kinds of FLS's without significant change of their structure. Non-singleton type-1 FLS consider input measurement as type-1 fuzzy number. Therefore the overshoot of the closed-loop system was smaller. The singleton type-2 FLS incorporate the uncertainty into fuzzy sets in antecedent and consequent. In this case the overshoot of the system was again smaller and the oscillations were damped. The last two FLS with type-1 and type-2 non-singleton fuzzification were even better. We quantified the performance of the systems with least squares error. All results are in Figure 11.

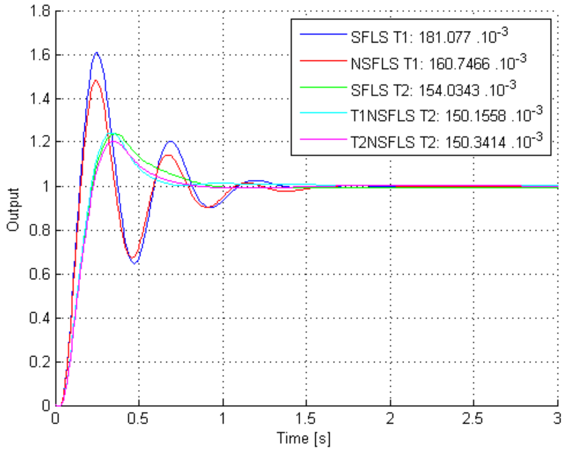

Figure 11: Closed loop system with all mentioned FLS

6 Conclusion

In this paper we presented Networked Control Systems and briefly described the problems that NCS imply. We showed that network induced delays could worsen the closed loop system performance. These input and output uncertainties can be modeled by FLS. We followed the work of professor Mendel [2] and designed five FLS to compare the performance of different types of FLS in NCS. The non-singleton type of the well known FLS shows some improvements but significant results can be gained by using of type-2 fuzzy sets. Type-2 fuzzy sets present new framework of FLS and show promising results in dealing with the uncertainties. In our work we chose type-2 fuzzy sets by our choice. From singleton type-1 FLS we got the input and output membership functions and then we have widen them by uncertain mean value. Even more interesting and precise results can be accomplished by the use of adaptive control strategies.

Acknowledgement

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