DESIGN OF DISCRETE CONTROLLER IN MATLAB

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Abstract

The aim of this paper is a design of discrete controller for control of linear continuous systems using chosen methods. Different design methods of discrete controllers are used for different types of continuous systems. These methods are compared in terms of quality of control. The design of discrete controller is implemented in MATLAB/Simulink (Graphical User Interface - GUI), where user can design, compare and verify different controllers applying proposed methods.

1 Introduction

Given the flexibility of a digital computer, digital control algorithms need not to be restricted to discrete versions of analog designs. In particular, it is possible to formulate controllers in such a way, that under ideal conditions they will produce desired closed loop response. This set of notes introduces some of these digital controller design methodologies. The aim of this paper is to describe the principles of control loop synthesis for control of a linear continuous systems using chosen methods.

There are two possibilities to get a discrete controller:

First procedure:
- obtain a discrete-time plant model (by discretization),
- design a discrete-time controller.

Second procedure:
- design a continuous-time controller,
- convert the continuous-time controller to discrete time.

In this paper, two groups of methods based on the first approach are chosen.

The first group contains the following methods: Inverse Dynamics method, Dead Beat and Dead Beat with constraint on manipulated variable.

Second group includes methods based on algebraic theory: Stable Time-Optimal Control (TOC), Finite TOC, Constrained Finite TOC, FeedForward and Quadratic Controller.

These methods are compared in terms of quality of control. An application is created by MATLAB/Simulink (Graphical User Interface - GUI), where users can design, compare and verify different controllers using proposed methods. The time responses of variables (controlled variable, reference variable, error and manipulated variable) are depicted. The practical use of this GUI like appropriate tool is for study matters concerning synthesis of control.

Continuous transfer function of control system is given by the following equation:

$$G(s)=\frac{b_m s^m + b_{m-1}s^{m-1}+\ldots+b_0}{a_n s^n + a_{n-1}s^{n-1}+\ldots+a_0}\,e^{-Ds} \qquad (1)$$

where b_i (i=0,1,…,m) are the parameters of numerator, a_j (j=0,1,…,n) are the parameters of denominator ($m<n$) and D is time delay. All of these parameters are entered by user, after choice of one of the design methods, parameters of discrete controller are calculated. These methods are based on discrete transfer function of control system $G(z)$. $G(z)$ refers to the Z-transform of the zero-order-hold device in series with the system being controlled. Discrete transfer function of control system is:

$$G(z)=\frac{b_{1d}z^{-1}+b_{2d}z^{-2}+\ldots+b_{nd}z^{-n}}{1+a_{1d}z^{-1}+a_{2d}z^{-2}+\ldots+a_{nd}z^{-n}}\,z^{-k} \qquad (2)$$

where b_{id} are parameters of numerator, a_{jd} are parameters of denominator and $k=D/T$, T is the sample time.

The designed controller affects the system dynamic. With the GUI created, influence of controller parameters on the closed loop step response can be seen. Four major characteristics of the closed loop step response are depicted: rise time, overshoot, settling time and steady-state error.

2 Discrete Controller

The discrete controller is used in a closed loop unity feedback system according to Fig. 1.

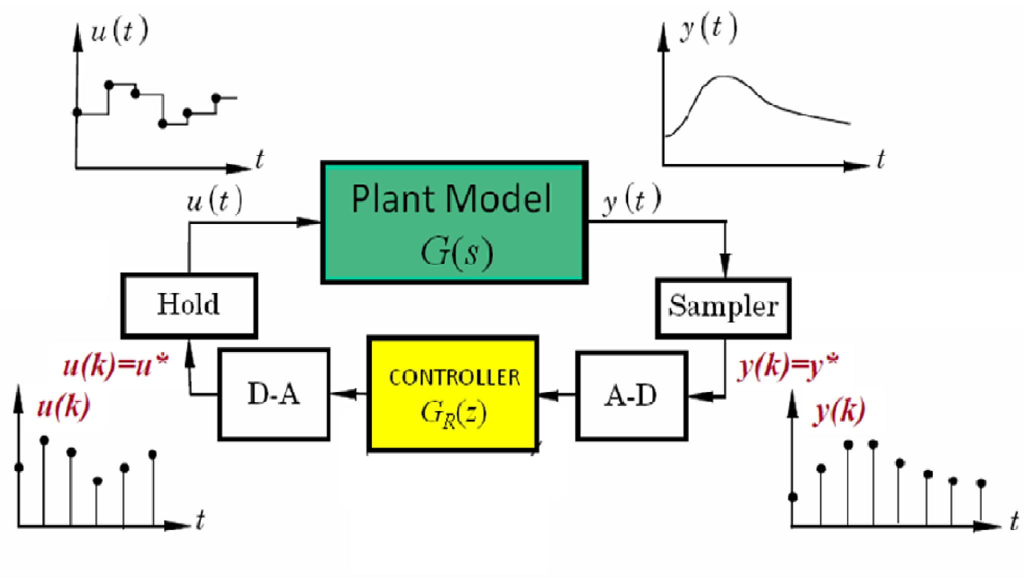

Figure 1: Block scheme of discrete closed loop control system

Discrete-data control systems have two unique features: the signals in these systems either are in the form of pulse trains or are digitally coded, and the controlled processes often contain analogue components [4]. For instance, an analogue plant model can be controlled either by a controller that outputs analogue signals or by a digital controller that outputs digital data. In the latter case, an interface such as a digital-to-analogue (D/A) converter is necessary to couple the digital component to the analogue device. Input and output of the discrete-data system can be represented by a number sequences with the numbers separated by the sampling period T. For linear operation, the D/A converter can be represented by a sample-and-hold device, which consists of a sampler and a data-hold device. The sample-and-hold that is most often used for the analysis of discrete-data systems consists of an ideal sampler and a zero-order-hold device.

Algorithms of design of discrete controllers for each of the mentioned methods in introduction of this paper are described in detail in [1].

3 Case Study

Discrete controller is tested in electric furnace temperature control, manipulating changes of power requirement. The actual temperature is sensed, whereby the control algorithm modulates the furnace power requirement. Block scheme of electric furnace control is on Fig. 2.

Model of electrical furnace is described by transfer function:

$$G(s) = \frac{b_0}{a_2 s^2 + a_1 s + a_0} e^{-Ds} = \frac{0.15}{s^2 + 1.1s + 0.2} e^{-1.5s} \qquad (3)$$

For this transfer function (the model of the electrical furnace) discrete controllers are designed by various methods, which are compared in terms of the quality of control.

The continuous-discrete problem (Fig. 1) is first converted to a discrete problem. Pulse transfer function $G(z)$ of the model is determined, considering the D/A converter and hold element together with the plant transfer function $G(s)$. Sampling time T has to be given.

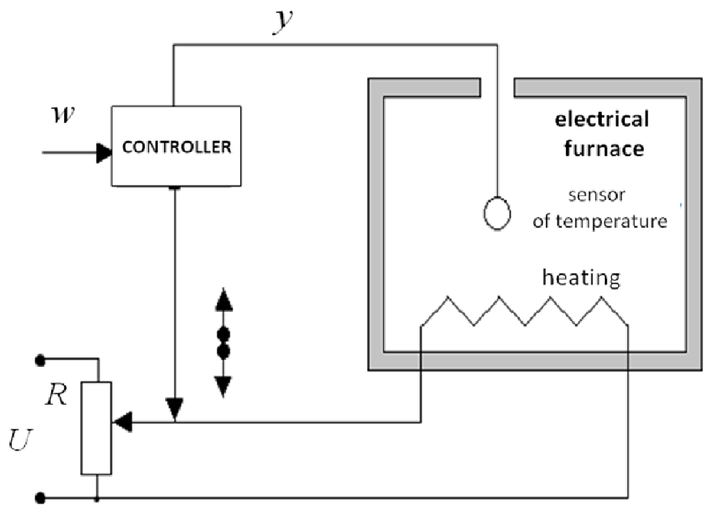

Figure 2: Block scheme of electric furnace control

Discrete transfer function of electrical furnace for T=1.5 s is:

$$G(z)=\frac{0.101z^{-1}+0.0584z^{-2}}{1-0.9795z^{-1}+0.1921z^{-2}}z^{-1}=\frac{0.101z^{-2}+0.0584z^{-3}}{1-0.9795z^{-1}+0.1921z^{-2}} \tag{4}$$

4 Design of discrete controller

Electrical furnace has been controlled with different types of discrete controller using the following methods: Inverse Dynamics method, Dead Beat and Dead Beat with constraint on manipulated variable, Stable Time-Optimal Control (TOC), Finite TOC, Constrained Finite TOC, FeedForward and Quadratic Controller. Detailed controller parameters design for all the above methods are introduced in [1], [2], [3].

The transfer functions of discrete controllers for individual methods are shown in Tab.1.

Table 1: TRANSFER FUNCTION OF DISCRETE CONTROLLER

METHOD	$G_R(z)$
Stable TOC	$\frac{9.901-9.6984z^{-1}+1.9021z^{-2}}{1+0.5782z^{-1}-z^{-2}-0.5782z^{-3}}$
Finite TOC	$\frac{6.2735-6.1449z^{-1}+1.205z^{-2}}{1-0.6336z^{-2}-0.3664z^{-3}}$
Constrained Finite TOC ($u\leq 6$)	$\frac{6-5.6035z^{-1}+0.8847z^{-2}+0.0525z^{-3}}{1-0.606z^{-2}-0.378z^{-3}-0.016z^{-4}}$
Feedforward	$\frac{9.901}{1+1.5577z^{-1}+0.5663z^{-2}}, \frac{-7.5972+1.863z^{-1}}{1+1.5577z^{-1}+0.5663z^{-2}}$
Quadratic Controller	$\frac{1-0.9795z^{-1}+0.1921z^{-2}}{0.101+0.0584z^{-1}-0.101z^{-2}-0.0584z^{-3}}$
Inverse Dynamics	$\frac{2.1026-2.0795z^{-1}+0.4323z^{-2}}{1-z^{-1}}$
Dead Beat	$\frac{6.2735-6.1449z^{-1}+1.2051z^{-2}}{1-0.6336z^{-2}-0.3664z^{-3}}$
Constrained Dead Beat ($u\leq 5.5$)	$\frac{5.5-4.6082z^{-1}+0.2997z^{-2}+0.1486z^{-3}}{1-0.555z^{-2}-0.3993z^{-3}-0.0452z^{-4}}$

Comparison of step responses of closed loop systems variable (y) under discrete control is shown in Fig. 3.

The comparison of time step responses of closed loop systems variable (u) under discrete control is shown in Fig. 4.

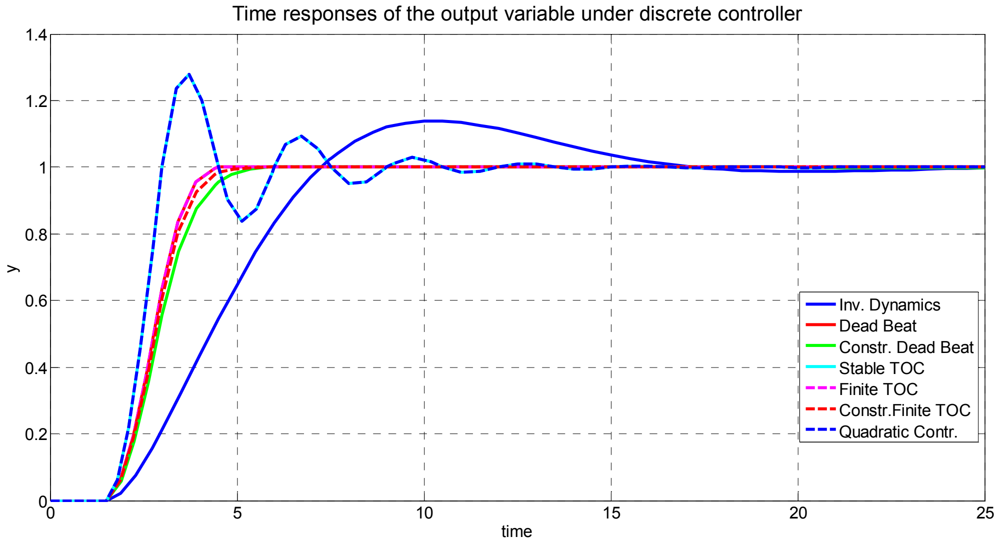

Figure 3: Step responses of closed loop systems variable (y) under discrete controller

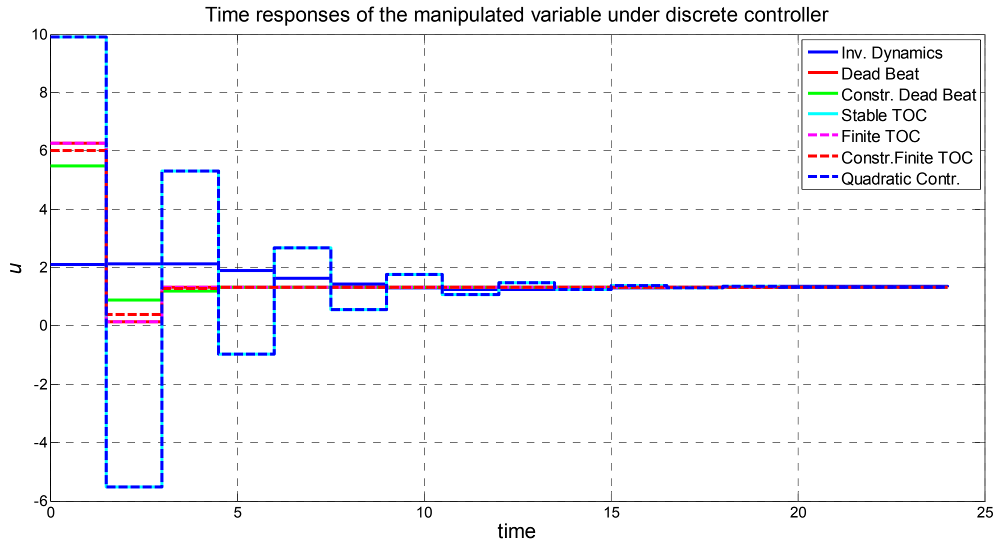

Figure 4: Step responses of closed loop systems variable (u) under discrete controller

Table 2: EVALUATION OF THE QUALITY OF CONTROL

METHOD	t_{reg} [s]	t_n [s]	η_{max} [%]
Stable TOC	10.0407	1.0264	27.8458
Finite TOC	4.2119	1.6961	0.0149
Constrained Finite TOC	4.4382	1.8109	0.0171
Quadratic Controller	10.0415	1.0264	27.8461
Inverse Dynamics	15.9726	3.9989	14.1963
Dead Beat	4.2119	1.6961	0.0149
Constrained Dead Beat	4.9267	2.0716	0.1994

Three major characteristics of closed loop step responses are evaluated in Tab. 2: settling time $t_{reg,}$ rise time t_n and overshoot $\eta_{\max}$.

Tab. 2 shows, that the most appropriate methods for the discrete control loop synthesis for the furnace process are the following methods: Dead Beat and Finite TOC. These methods ensure low value of settling and the rise times and although have reached the lowest values of the maximum overshoot.

5 Application of Design of Controller Parameters

Design of discrete controller (Fig. 5, 6) is implemented in MATLAB/Simulink (GUI). It is believed that the developed tool may be very useful for design and tuning of industrial controllers.

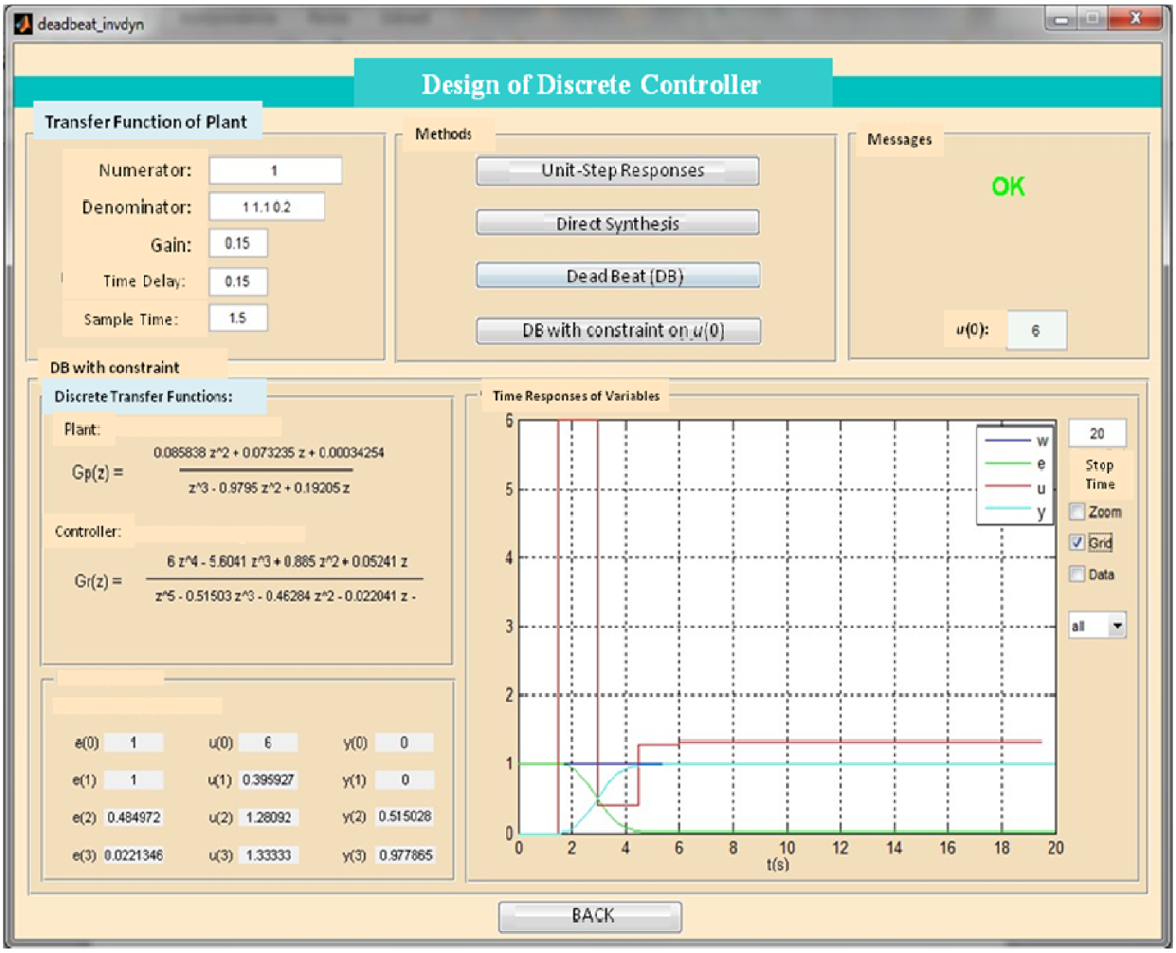

Figure 5: The Graphical Tuning Window1

GUI can accept any model plant transfer function. User can choose one of the above described methods for discrete controller design and compare it with other methods offered in GUI menu on the basis of the possibility of step response of closed control loop. The quality and stability of control with discrete controllers can be evaluated.

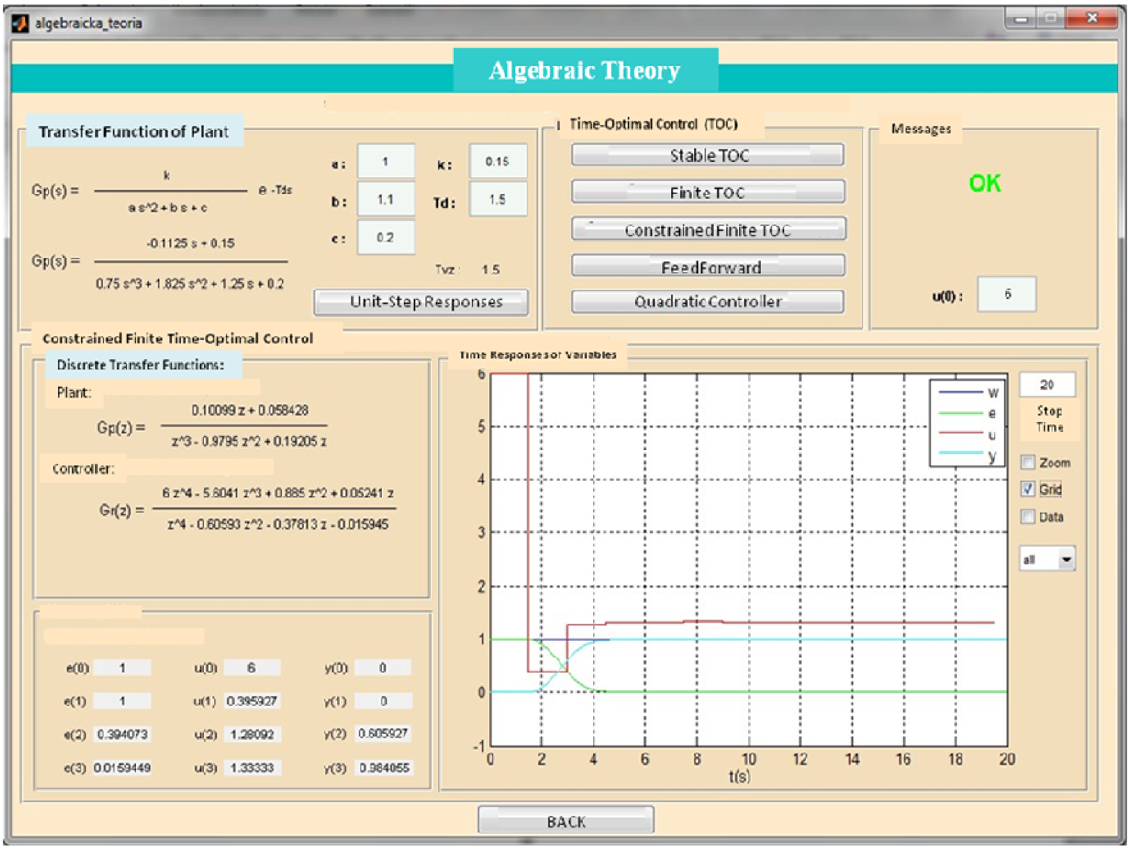

Figure 6: The Graphical Tuning Window2

6 Conclusion

GUI was created for educational purposes in subjects where the basics of automatic control are introduced. Students can see the influence of each of the control parameters on the quality and stability of the control.

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