Galton Board

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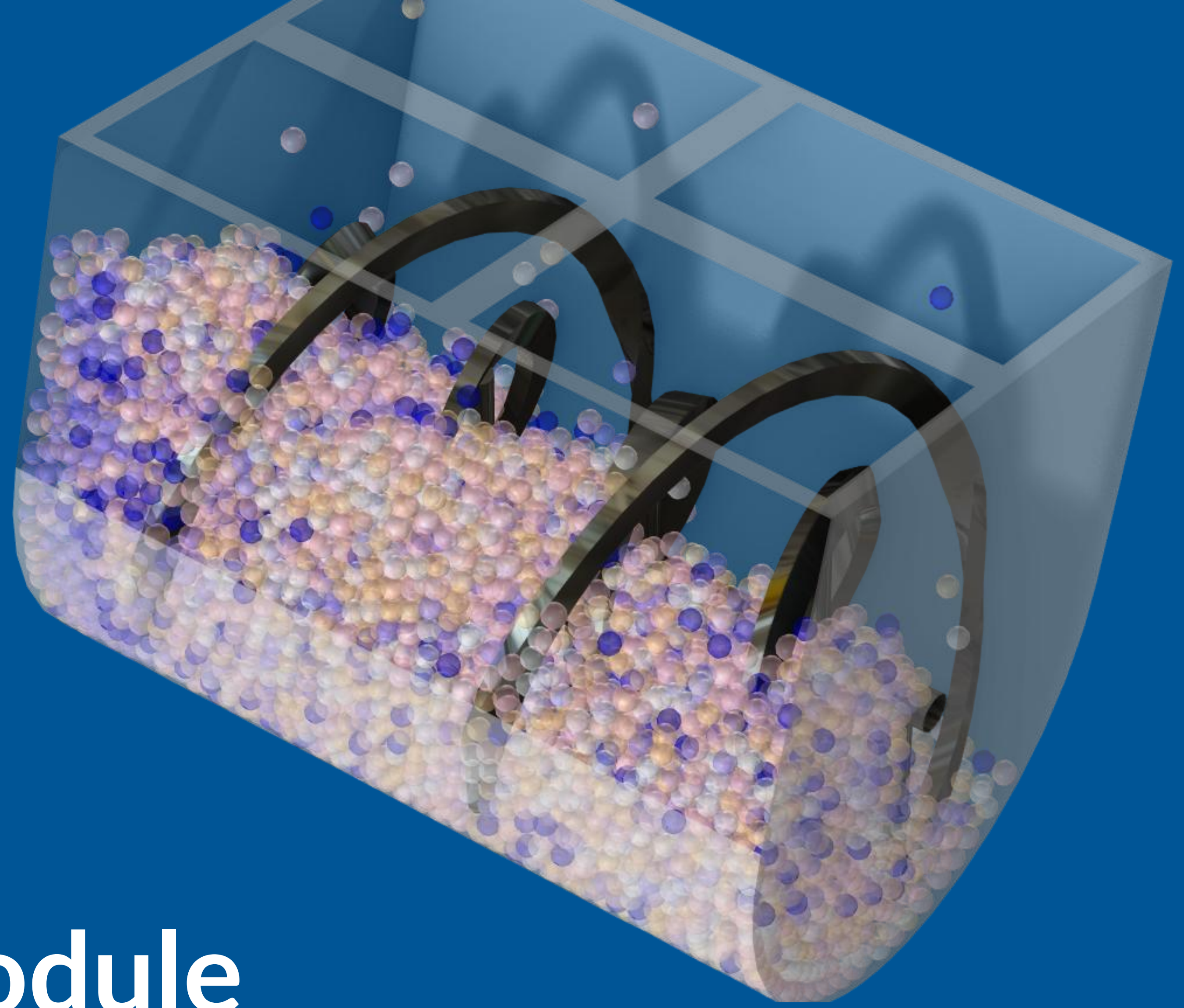

Granular Flow Module

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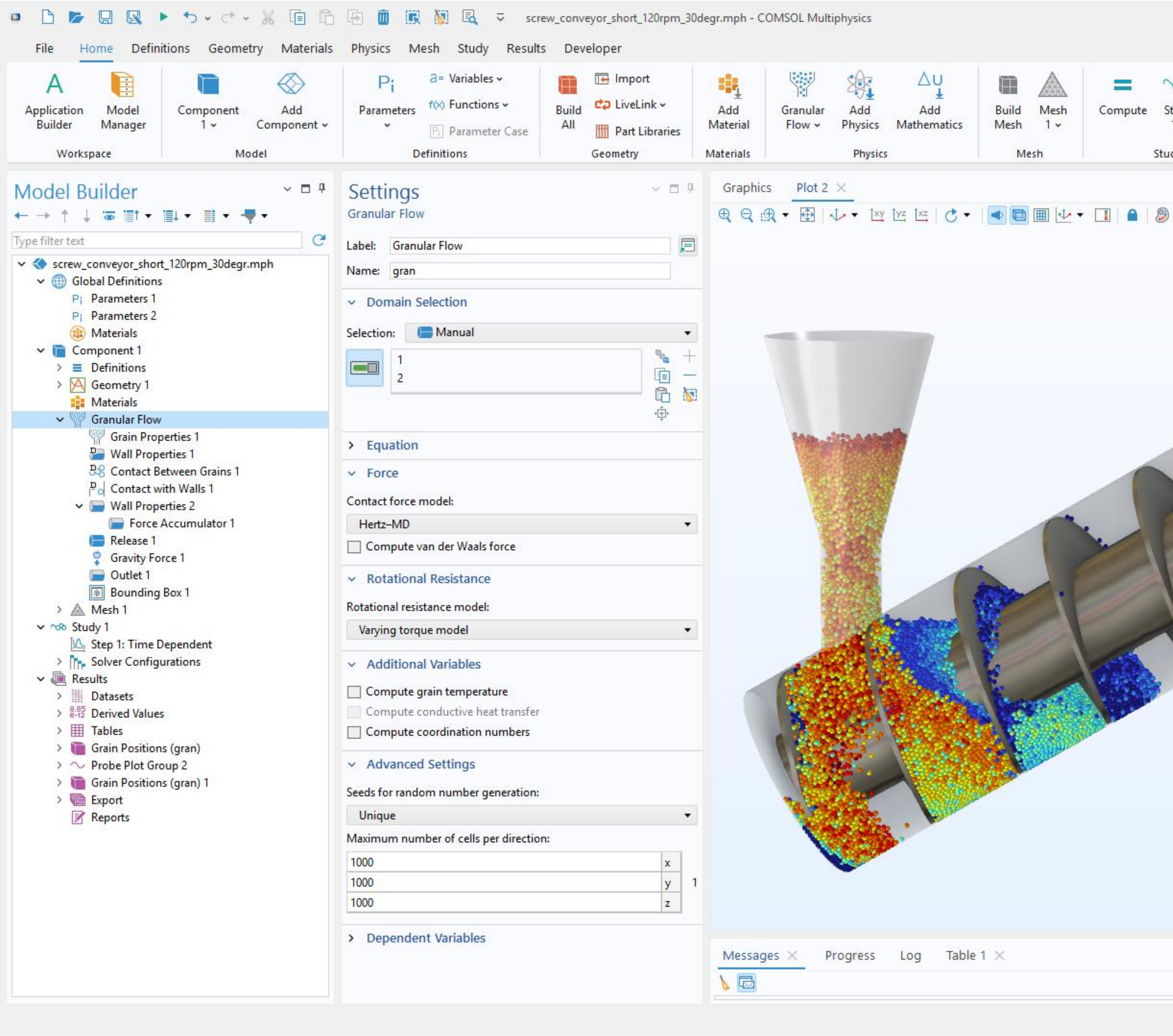
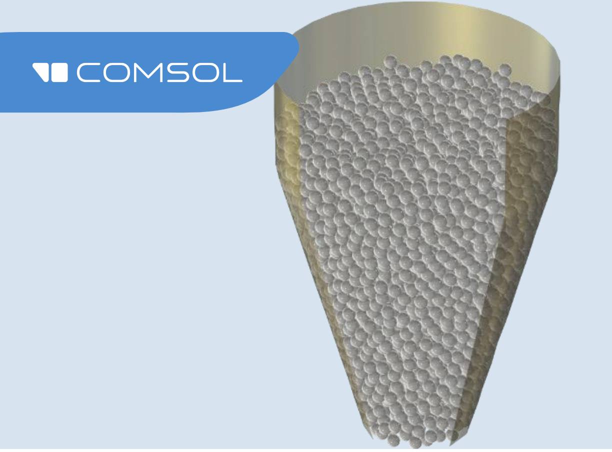
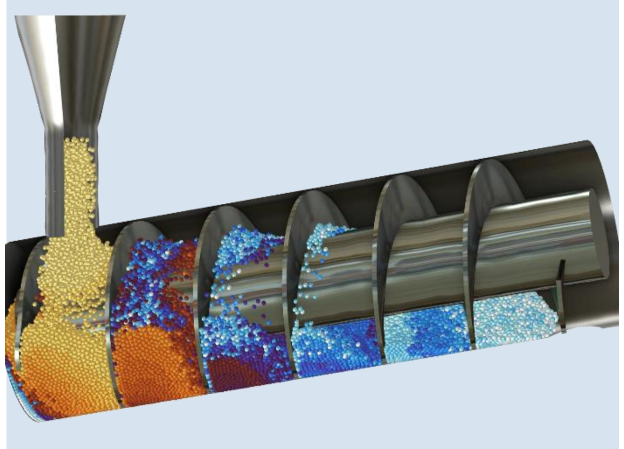
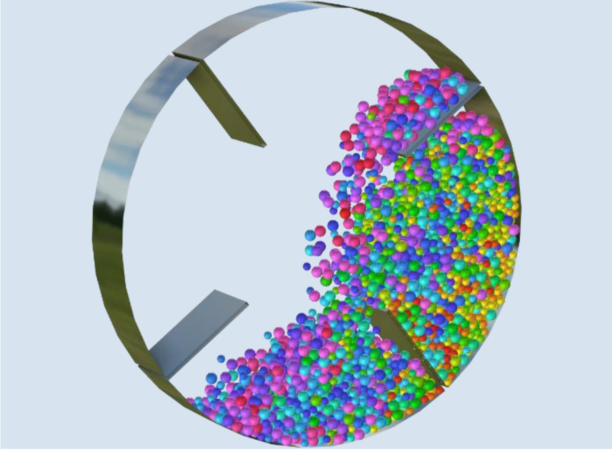
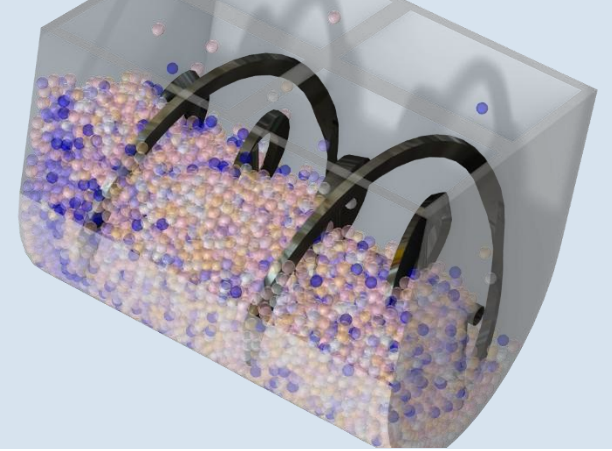
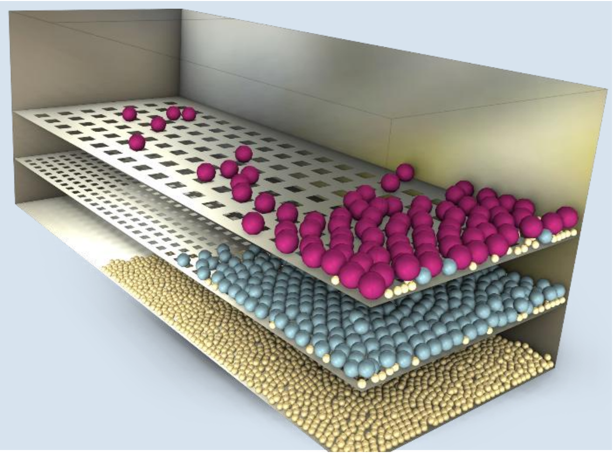
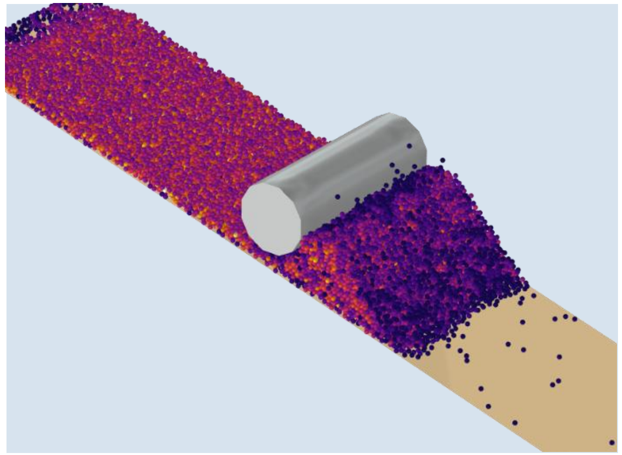
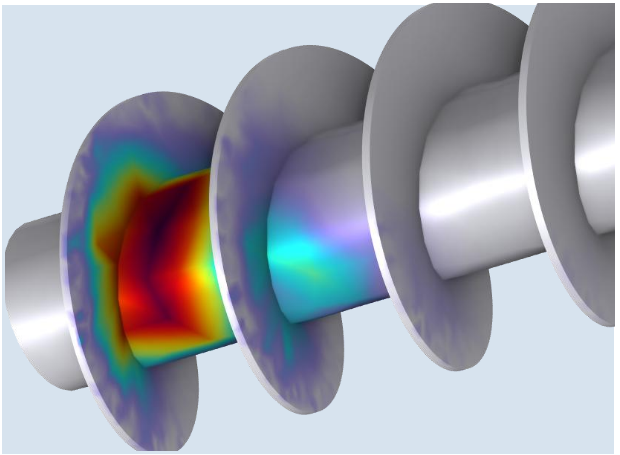
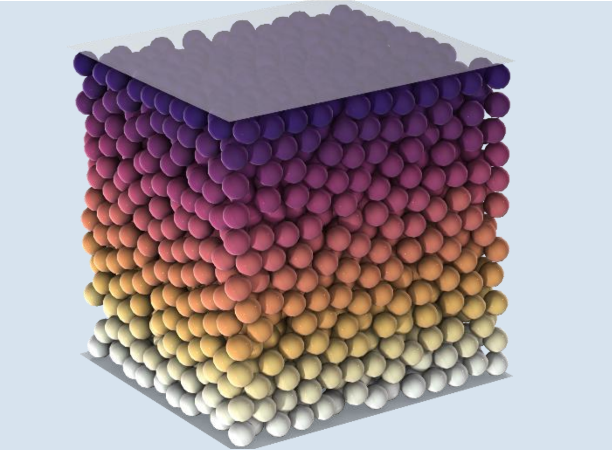
Granular Flow Module

- Simulate the bulk flow of granular materials
- Grains are discrete entities with finite size and shape
- Track position and velocity of individual grain in an external force field
- Compute contact forces between grains and between grains and walls using discrete element method (DEM)
- Track orientation, angular velocity and tangential displacement due to torque



Application Areas

Some of the common application areas are filling/packing, emptying, transporting, mixing, separating and filtering, heat transfer.

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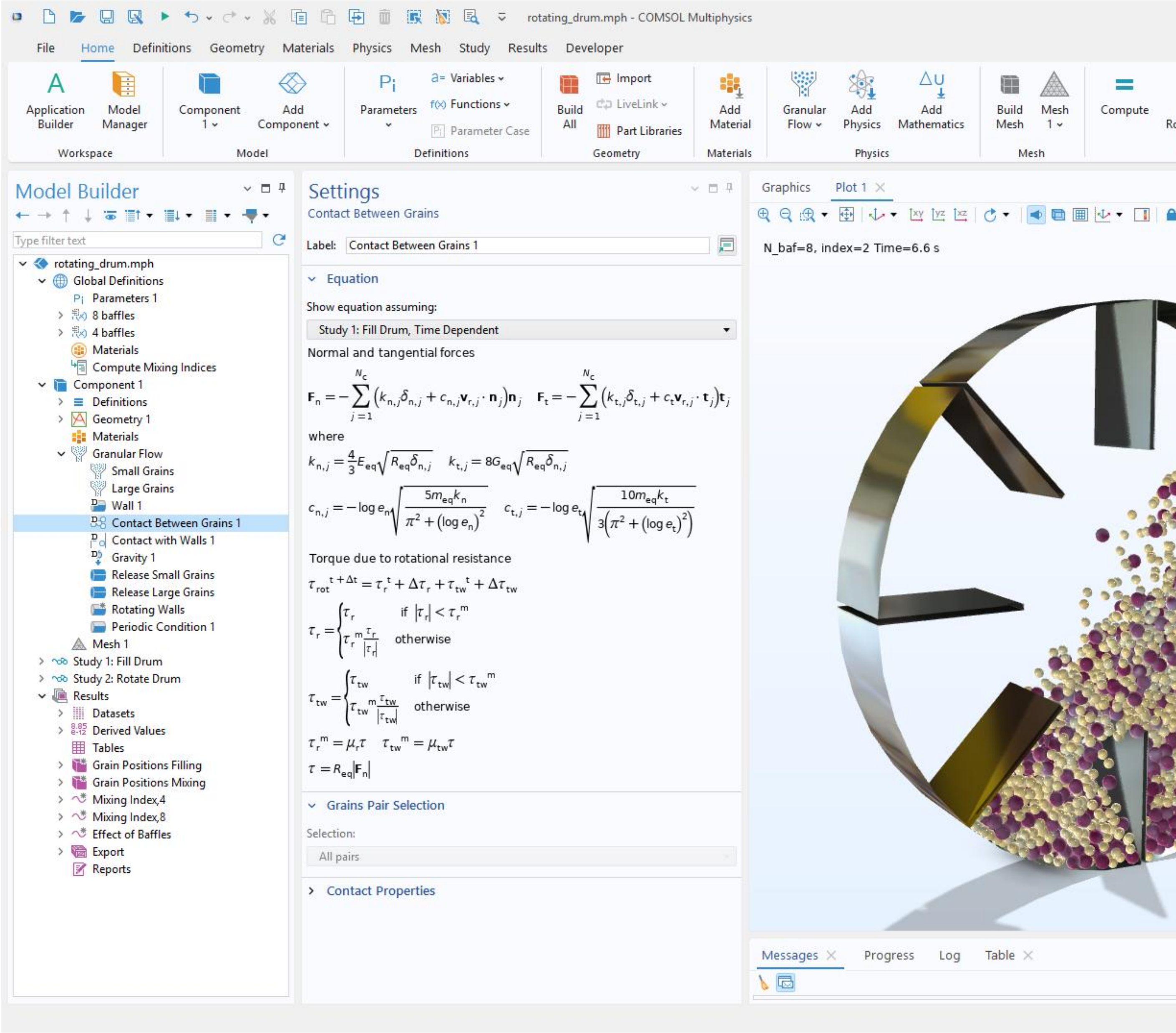
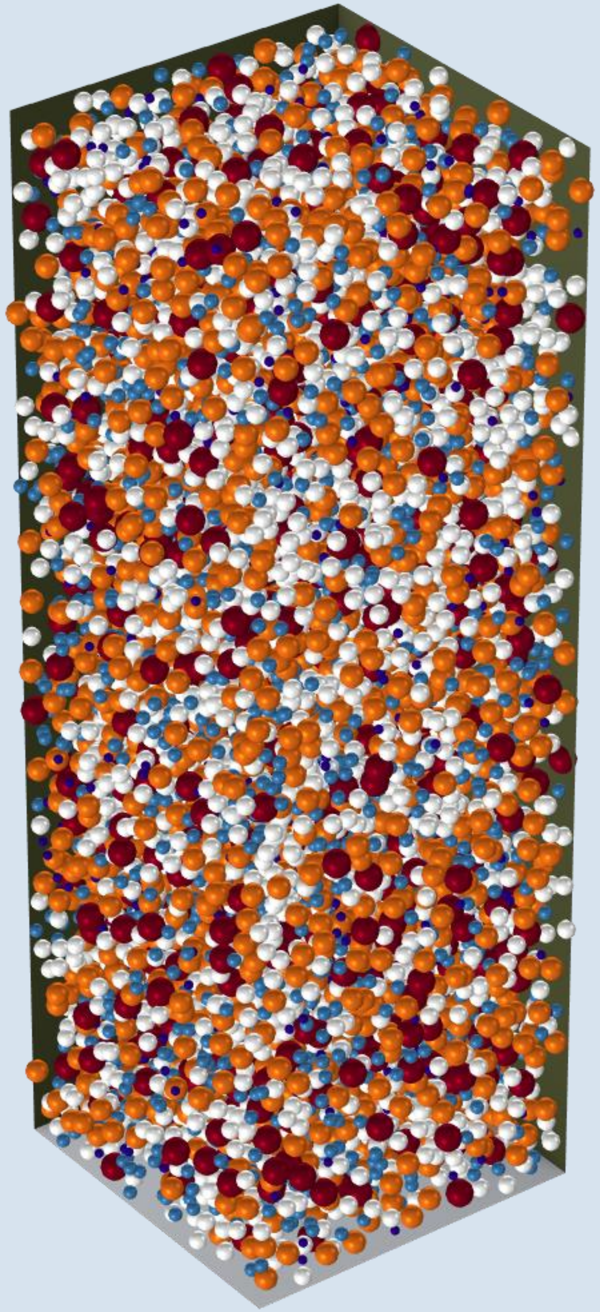
Granular Flow Module

- Various types of resistances to the motion
  - Resistance due to sliding friction
  - Resistance torque opposing rolling
  - Resistance torque opposing twisting
- Damping of forces and torques
- Option to add adhesive and van der Waals forces to the contact forces
- Simulate the effect of moving boundaries on the granular flow
- Multiple species of grains and different types of walls



Multiple Species Initialization

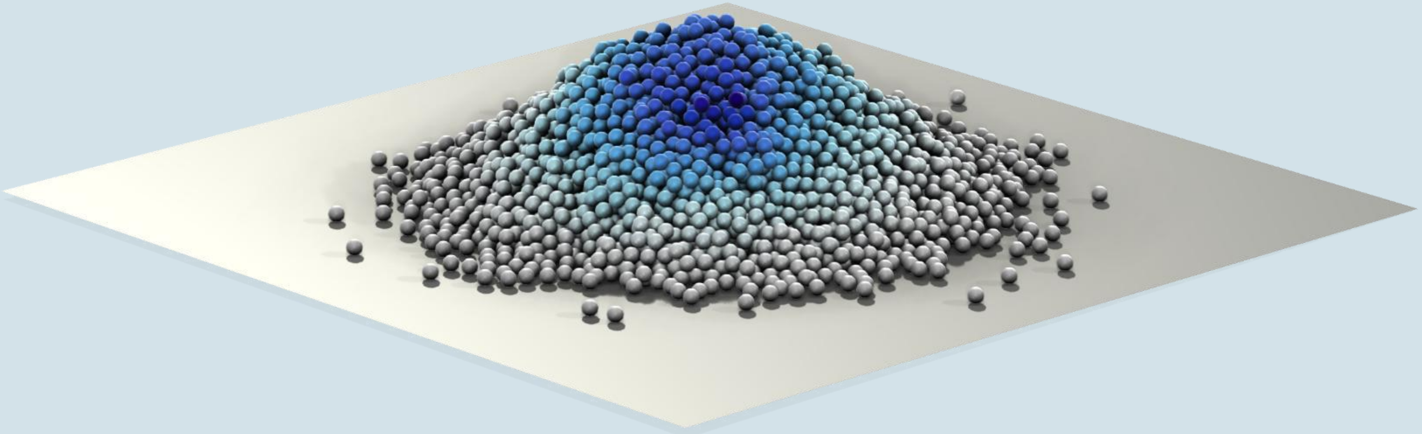
- Release multiple types of grains using a single release feature
- Options to choose for the distribution of released grains
  - Number of grains
  - Number fraction
  - Mass fraction
  - Volume fraction
- Multiple release times



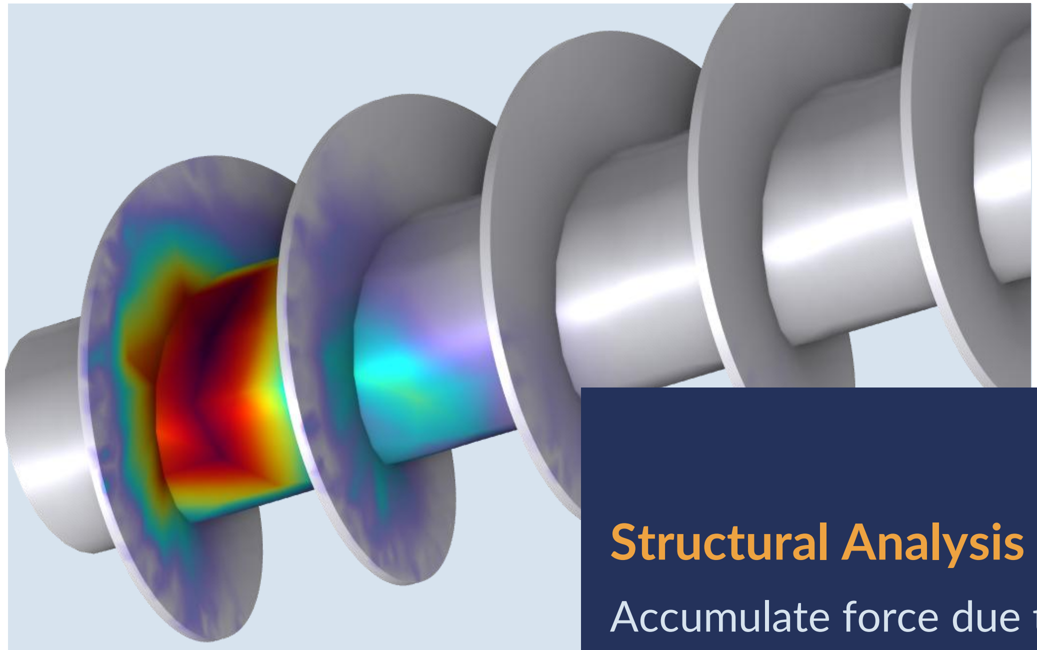
Multiple species of grains initialized using a Release feature

Boundary Conditions

- *Wall*
  - Accumulate force on the boundary elements
  - Heat transfer
- *Outlet*
  - Disappear
  - Pass through
- *Periodic Condition*

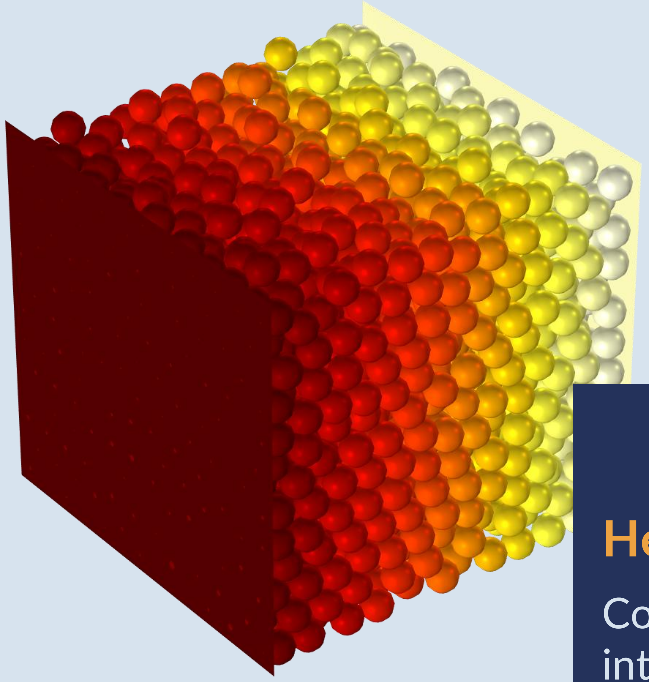


Multiphysics Couplings with Granular Flow



Structural Analysis

Accumulate force due to contact between grain and wall on the boundary elements and use it as source for the structural analysis

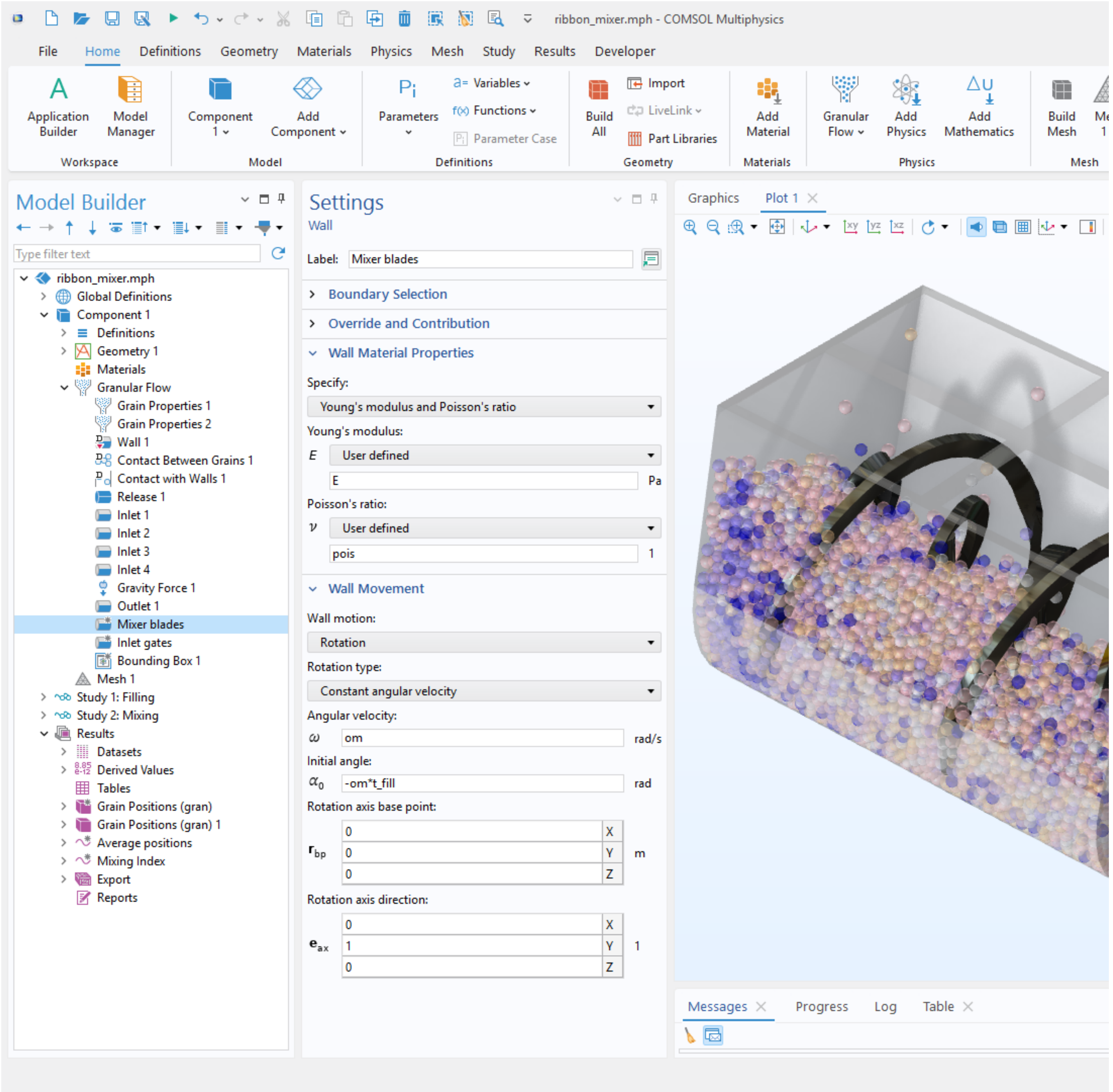


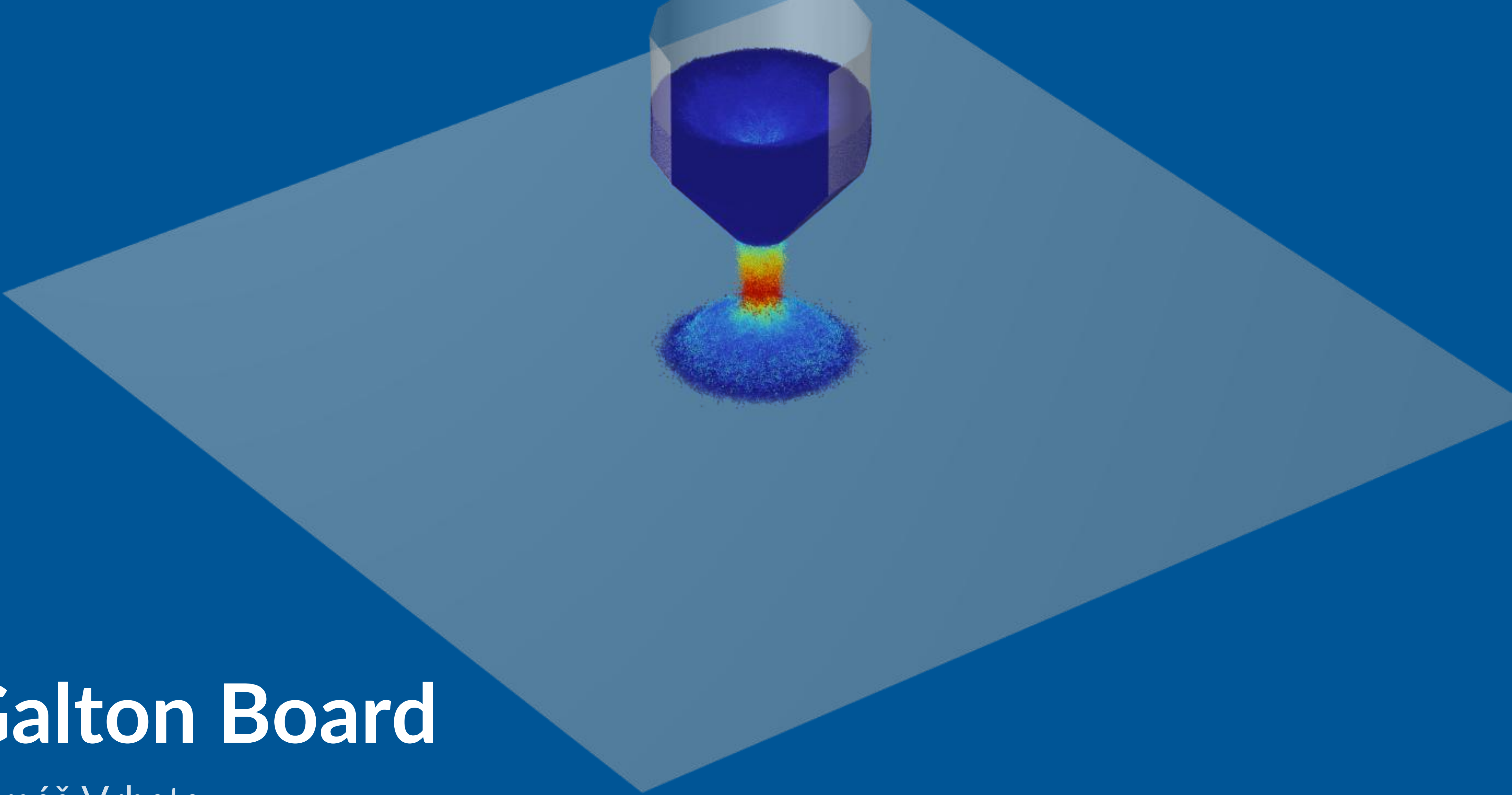
Heat Transfer

Couple with heat transfer interfaces to compute surface temperature and use it to simulate heat transfer in granular system

Essential Features in a Granular Flow Model

- At least one:
  - *Grain Properties* node
  - *Wall* node
  - *Contact between Grains* node
  - *Contact with Walls* node
  - Release feature
- Applied forces
- Optional:
  - Use moving boundaries
  - Accumulate force on wall elements
  - Compute grain temperature





Galton Board

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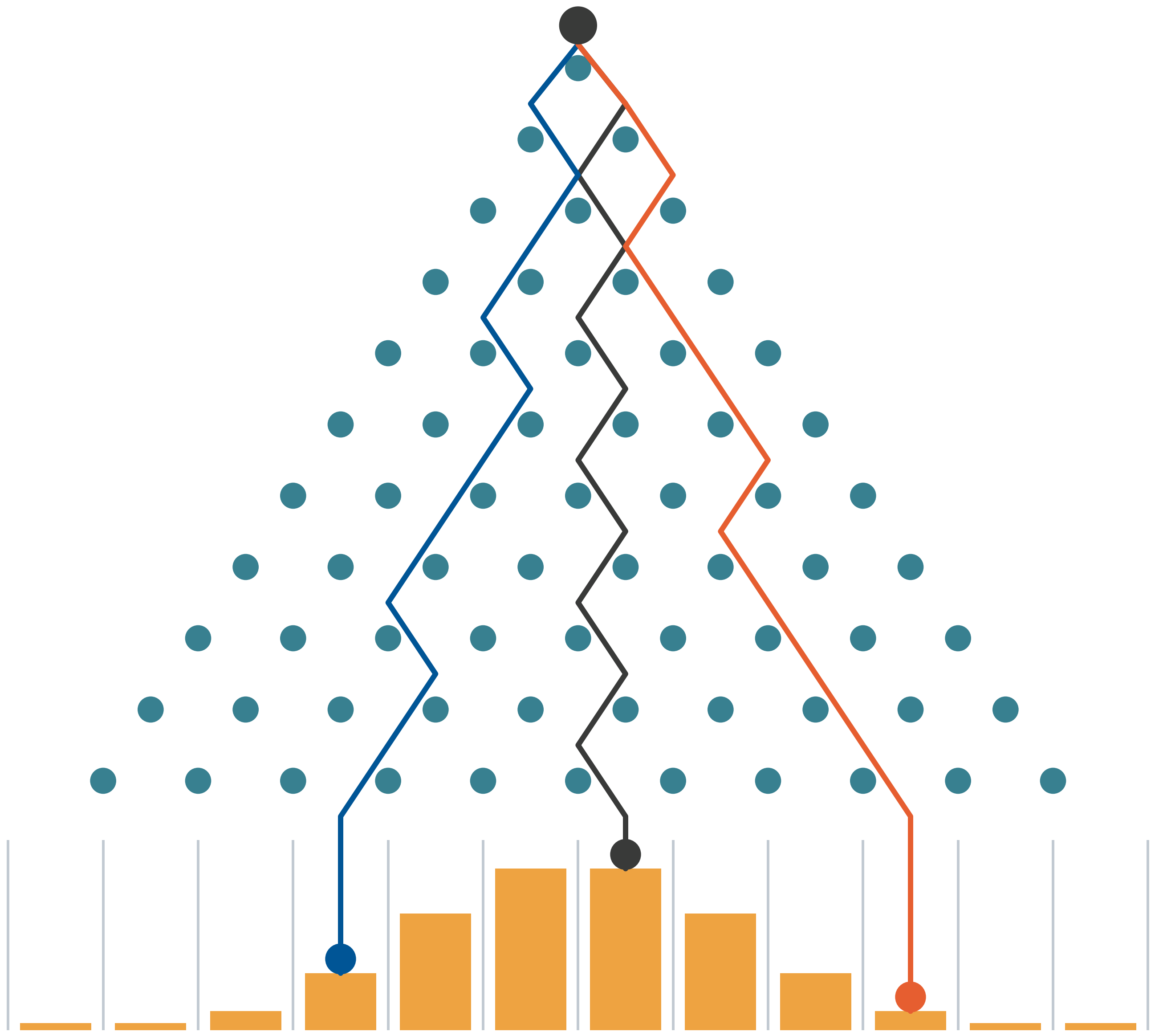
How randomness produces a normal distribution

- Balls fall through a grid of pins, bouncing randomly left or right in each row; the column heights at the bottom trace out a bell curve.



How it works

1. **A random step in every row**

   At each pin the ball bounces left or right, roughly 50/50 — like a coin toss.

2. **Thousands of tiny balls**

   ~1,000 steel balls 1 mm in diameter are poured onto the top through a hopper.

3. **Collection in bins**

   The heights of the columns collected in the bins below trace the resulting distribution.

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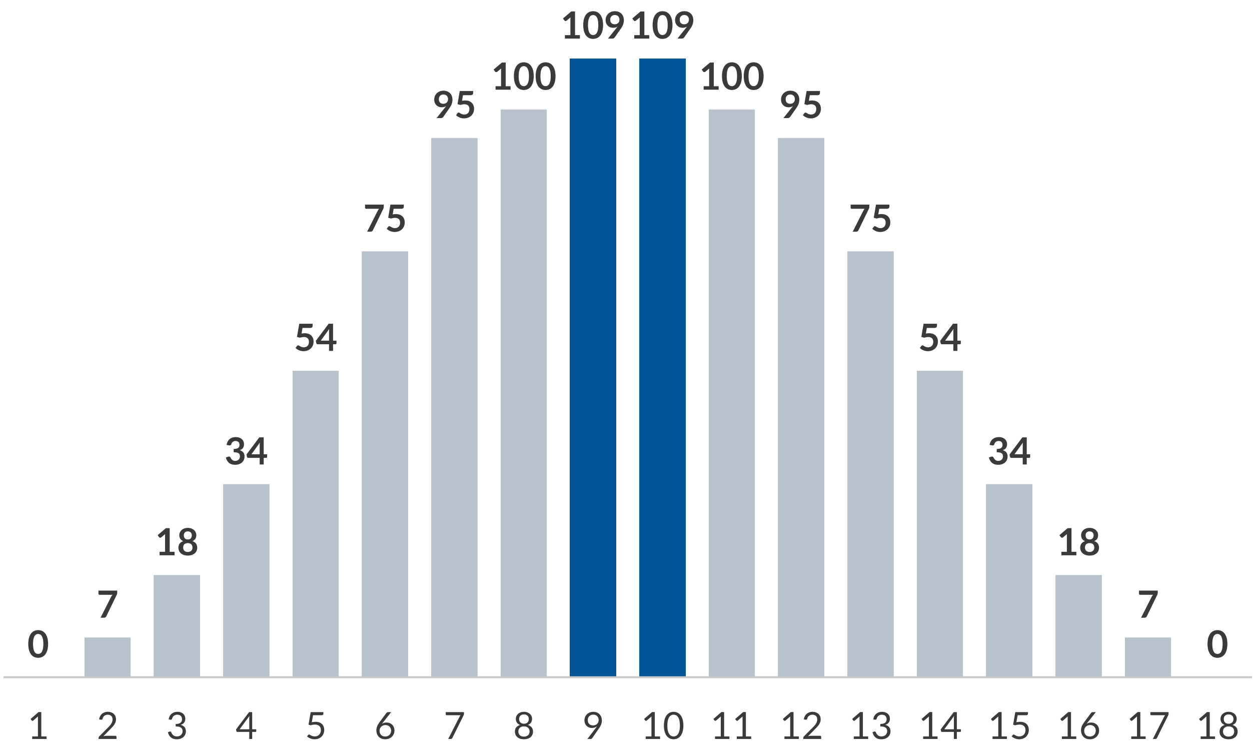
Result: bell curve vs. theory

The bell curve is the central limit theorem

- The balls pile up in the bins along a normal (Gaussian) curve.
- Each pin is a 50/50 step, so the bin totals follow a binomial distribution that approaches the normal as the number of rows grows.

Expected distribution across the 18 bins

Illustrative · ~1000 balls · theoretical



Showcase: Galton Board

- **Problem type**

  Granular flow – discrete element method (DEM), 2D, time-dependent.

- **Goal**

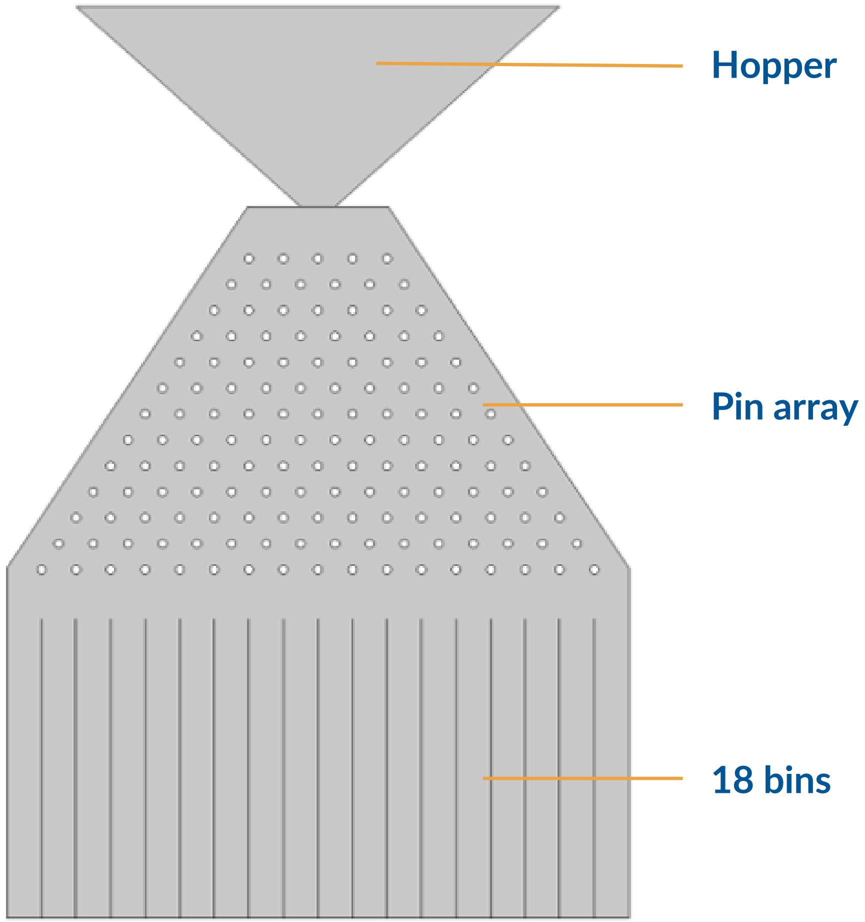
  Show how random bounces of ~1000 balls produce a binomial, and hence normal, distribution.

- **Monitored quantities**

  Mean position ave = gran.ave(qx) and standard deviation sigma_a across the bins.

Model Setup — Geometry

- **Geometry**
  - Hopper – pours in the balls
  - Pin array – triangular layout (even and odd rows)
  - 18 bins – collect the balls

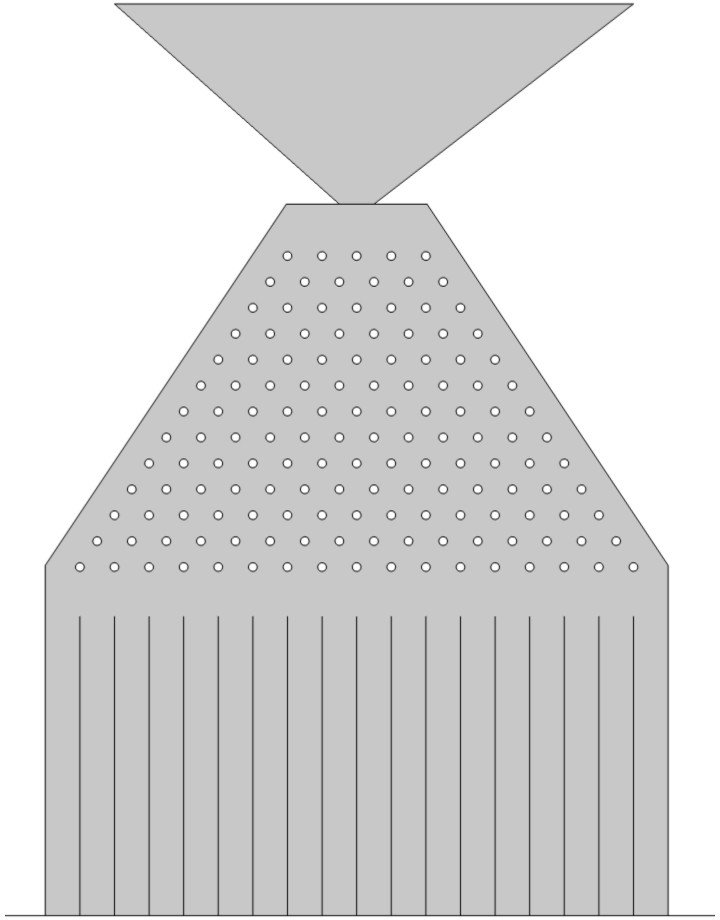


Model Setup — Physics

- **Geometry**

  Hopper, triangular pin array, 18 bins.

- **Granular Flow**
  - Diameter 1 mm, 1000 balls
  - Gravity
  - Contact between grains
  - Contact with walls
  - Contact model with friction
  - Release: all balls at once, random position

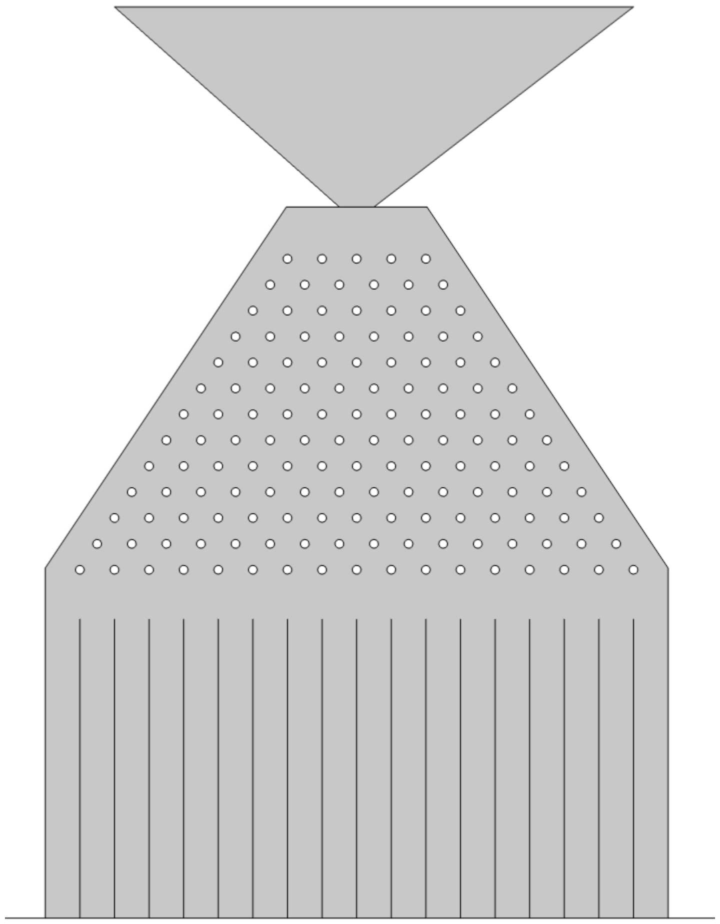


Model Setup — Mesh & Study

- **Geometry**

  Hopper, pin array, 18 bins.
- **Granular Flow**

  Grains, gravity, contacts, release.
- **Mesh & Study**
  - Mesh: free quadrilateral
  - Study: two steps
    - Step 1 — fill the balls into the hopper
    - Step 2 — solve the balls falling through the board

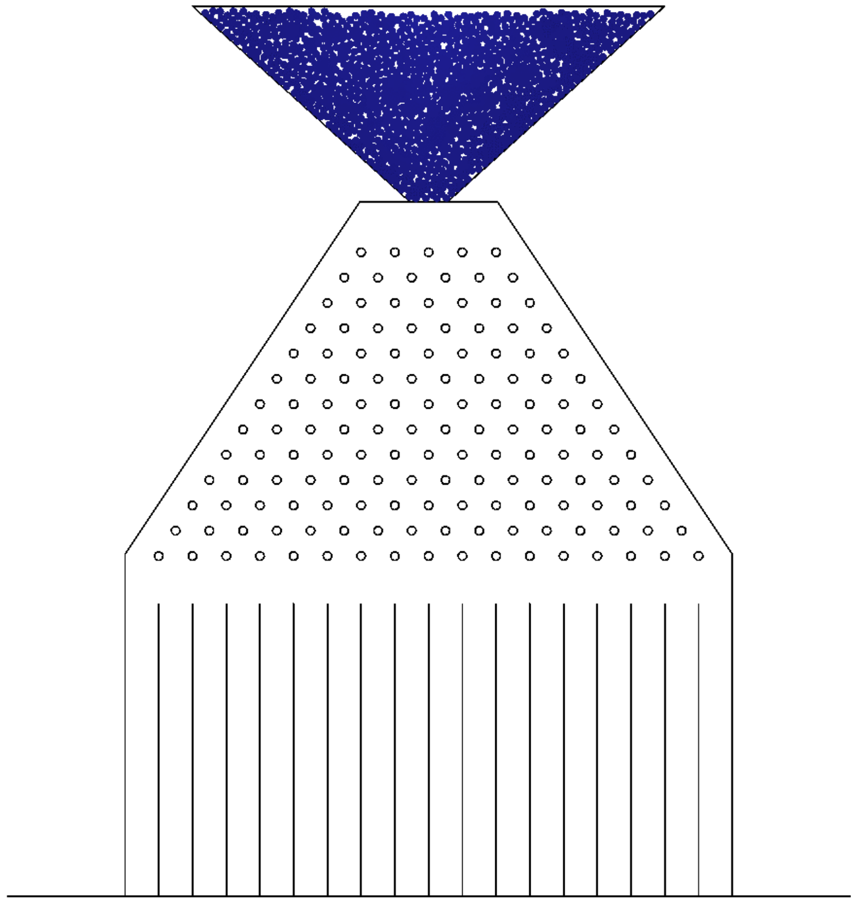


Results

- **Result**

  The column heights in the bins trace a bell-shaped distribution.

- **Statistics**
  - Mean position: ave = gran.ave(qx)
  - Standard deviation: sigma_a



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File

New

Model Wizard

Blank Model

Help Cancel ☑ Show on startup

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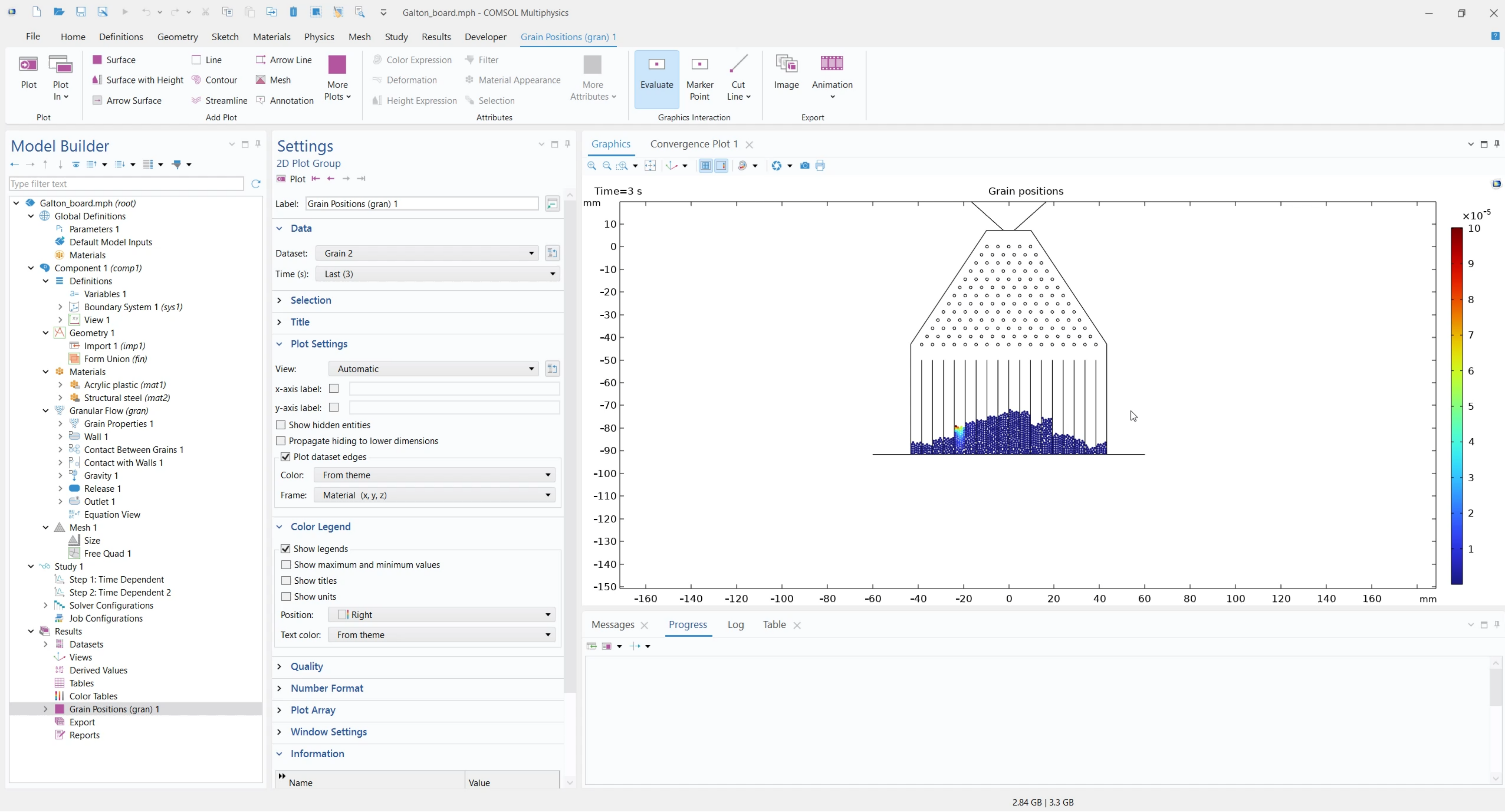
Galton_board.mph - COMSOL Multiphysics
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Definitions
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Sketch
Materials
Physics
Mesh
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Results
Developer
Grain Positions (gran) 1
Plot
Plot In
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Surface with Height
Arrow Surface
Line
Contour
Streamline
Arrow Line
Mesh
Annotation
More Plots
Add Plot
Color Expression
Deformation
Height Expression
Filter
Material Appearance
Selection
More Attributes
Attributes
Evaluate
Marker Point
Cut Line
Graphics Interaction
Image
Animation
Export
Model Builder
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Galton_board.mph (root)
Global Definitions
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Component 1 (comp1)
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Variables 1
Boundary System 1 (sys1)
View 1
Geometry 1
Import 1 (imp1)
Form Union (fin)
Materials
Acrylic plastic (mat1)
Structural steel (mat2)
Granular Flow (gran)
Grain Properties 1
Wall 1
Contact Between Grains 1
Contact with Walls 1
Gravity 1
Release 1
Outlet 1
Equation View
Mesh 1
Size
Free Quad 1
Study 1
Step 1: Time Dependent
Step 2: Time Dependent 2
Solver Configurations
Job Configurations
Results
Datasets
Views
Derived Values
Tables
Color Tables
Grain Positions (gran) 1
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Thank you for your attention

Any questions?