SPECTRAL ANALYSIS OF NON-STATIONARY SIGNALS USING ZOLOTAREV POLYNOMIALS

Spektrální analýza nestacionárních signálů s využitím Zolotarevových polynomů

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Abstract

An alternative method for analaysis of non–stationary signals is proposed. In the core of this method Zolotarev polynomials and functions derived from Zolotarev polynomials are used. Zolotarev polynomials can be considered as a generalisation of Chebychev polynomials [1]. Basic principles and some experimental results of this method are compared to short–time Fourier transorm and Wavelet transform as representatives of the two most dangerous competitors.

1. Introduction

Spectral analysis based on signal decomposition into a set of trigonometric functions is one of the most common signal processing method. Using well known transforms such as FT, DTFT or DFT, signal under analysis is converted from time–domain into frequency–domain. The superior rank of all FT–like transforms does lay in the fact that trigonometric functions are eigen functions for linear time–invariant (LTI) systems, most common type of systems and signals. During such kind of transform all information carried by signal waveform is converted into its frequency spectrum.

For non-stationary signals whose spectra change in time, a lot of methods has been proposed during last decades. Among all these methods, Short Time Fourier Transform (STFT) a Wavelet Transform (WT) seem to be most popular. STFT based on signal segmentation followed by some kind of Fourier transform suffers mainly from uncertainity relations when increasing time resolution is penalized by decreasing time resolution and vice versa. Wavelet transform tries to make an optimal compromise between time and frequency resolution by scaling time resolution according to frequency. The method proposed in this paper tries to show a possibly alternative method for non–stationary signal analysis based on functions derived from Zolotarev polynomials [1].

Zolotarev polynomials can be considered as generalized Chebychev polynomials. As Chebychev polynomials change into trigonometric functions under appropriate parametrizations, Zolotarev polynomials can be similarily parametrized so they can represent "generalized" trigonometric functions. Thus, these functions can be used for signal analysis.

2. Results

Figure 1 shows an exaple of non–stationary signal composed of two harmonic signal containing an abrupt phase change. The signal is represented by 64 samples only. On the same figure a spectrogram of this signal is shown. According to small signal lenght, parameters of STFT were chosen as follows – DFT size 16 samples, Hamming window, windows overlap 15 samples.

As can be seen, the time–frequency resolution is quite small. Using Zolotarev–based functions on the same signal, results shown in figure 2 can be obtained.

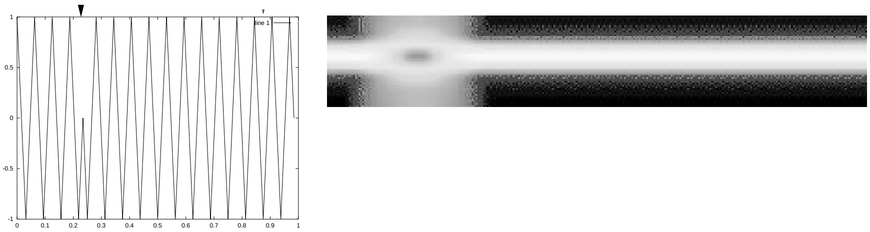

Figure 1: Testing signal containig phase discontinuity and its spectrogram, DFT size N=16, overlap 15 samples, Hamming window. Arrows represents the time localization of discontinuity.

It is clear that reached time–frequency resolution is much better compared to STFT and the discontinuity can be seen much clearly. Depending on parameters of Zolotarev–based functions time or frequency behavior of signal becomes more transparent.

Another example of features of Zolotarev based transform is shown at figure 3. A microphone generated glitch is displayed in time domain followed by its various transformations. The first one is its spectrum. According to used STFT parameters, the frequency resolution is quite good, however the time resolution is comparable to the glitch duration and therefore time location of the glich from its spectrogram is problematic. The time localization could be improved by shortening the lenght of used window however this step would obviously decrease the frequency resolution. Wavelet transform shows clearly time location of the glitch nevertheless the interpretation of this wavelet scale–time chart is rather difficult as one can say only that "something happend" at low scales. On the other hand, Zolotarev based transform preserves spectrum–like look and feel while the time localization of the glitch is improved. Further time localization improvement is possible through Zolotarev transform adjustement, but increasing time resulution results in the loss of spectrum similarity and, again, frequency interpretation (or resolution) decreases.

Figure 4 shows a piece of EKG signal as a typical representative of localy non–stationary signal. Fourier analysis of such kind of signal is rather difficult and therefore time–domain methods and algorithms often take place in such cases. However the third picture of figure 3 shows that processing by Zolotarev based transform can show spectrum–like picture with relatively good both frequency and time resolution.

Finally, an analyssis of a speech signal sampled at 8000 Hz is shown. Looking at the STFT one could believe that this sample of signal consists of three stationary parts. A formant

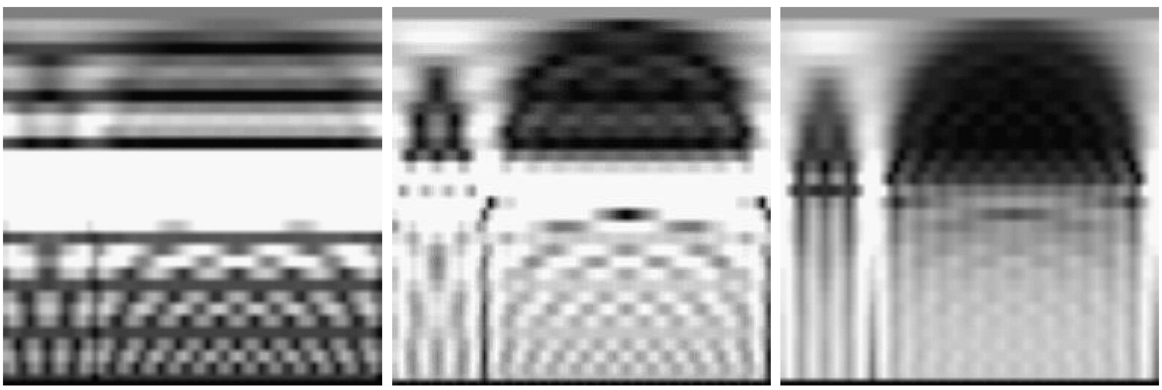

Figure 2: Analysis of signal from 1 using Zolotarev–based functions. Zolotarev peak value were chosen as follows $Z_{max} = 2,\ 10,\ 50$. Higher scales respond to lower frequency.

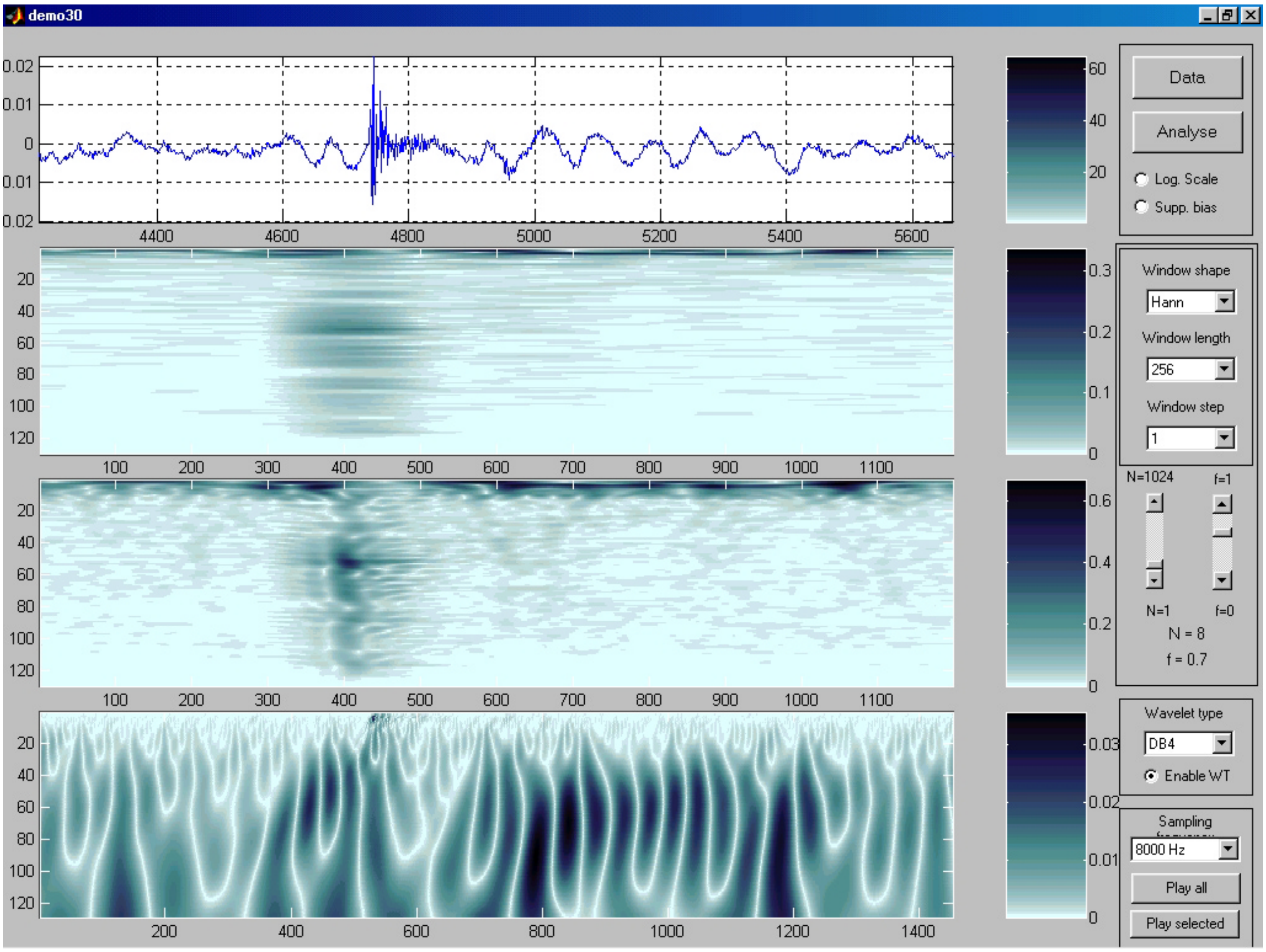

Figure 3: Time glitch. First subsection shows time domain, the second one spectrum, Zolotarev transformed signal follows and finally the same signal transformed using Wavelet transform. Parameters of all methods are shown on the right hand side of the figure.

frequency structure of speech can be seen clearly from STFT. Zolotarev transform preserves this structure, however it shows in addition quite clearly local non–stationary parts of the signal (light and dark bars around formant frequences). Lenghts of these bars re–computed to time duration result in times approximately 10 – 30 ms. This seems to be in quite good agreement with what is known about semi–stationarity of speech signals.

3. Conclusions

Research in this field is at its very beginning, however, some promising results have been already obtained. For further information, please don't hesitate and contact authors.

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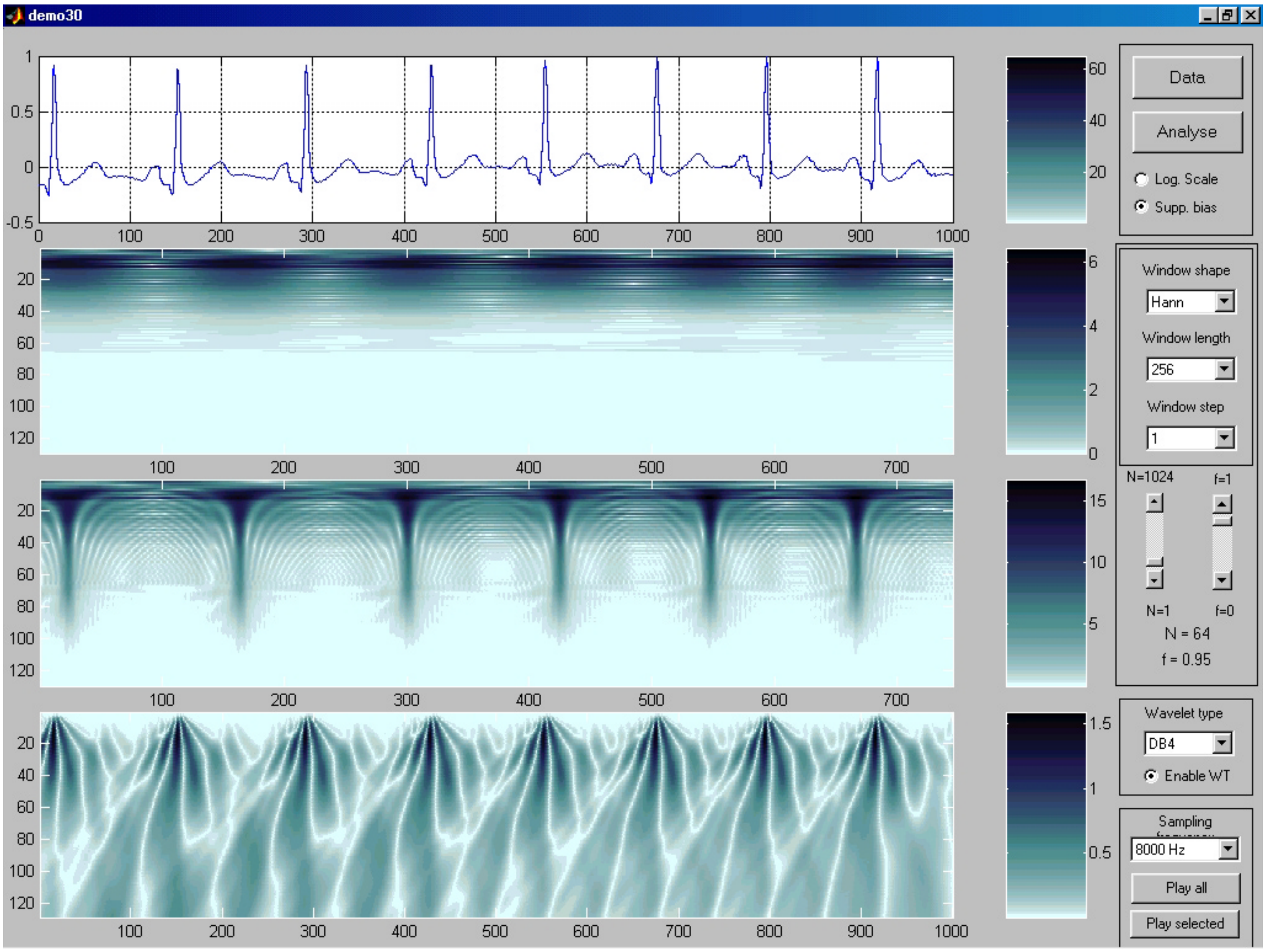

Figure 4: A piece of EKG signal. First subsection shows time domain, the second one spectrum, Zolotarev transformed signal follows and finally the same signal transformed using Wavelet transform. Parameters of all methods are shown on the right hand side of the figure.

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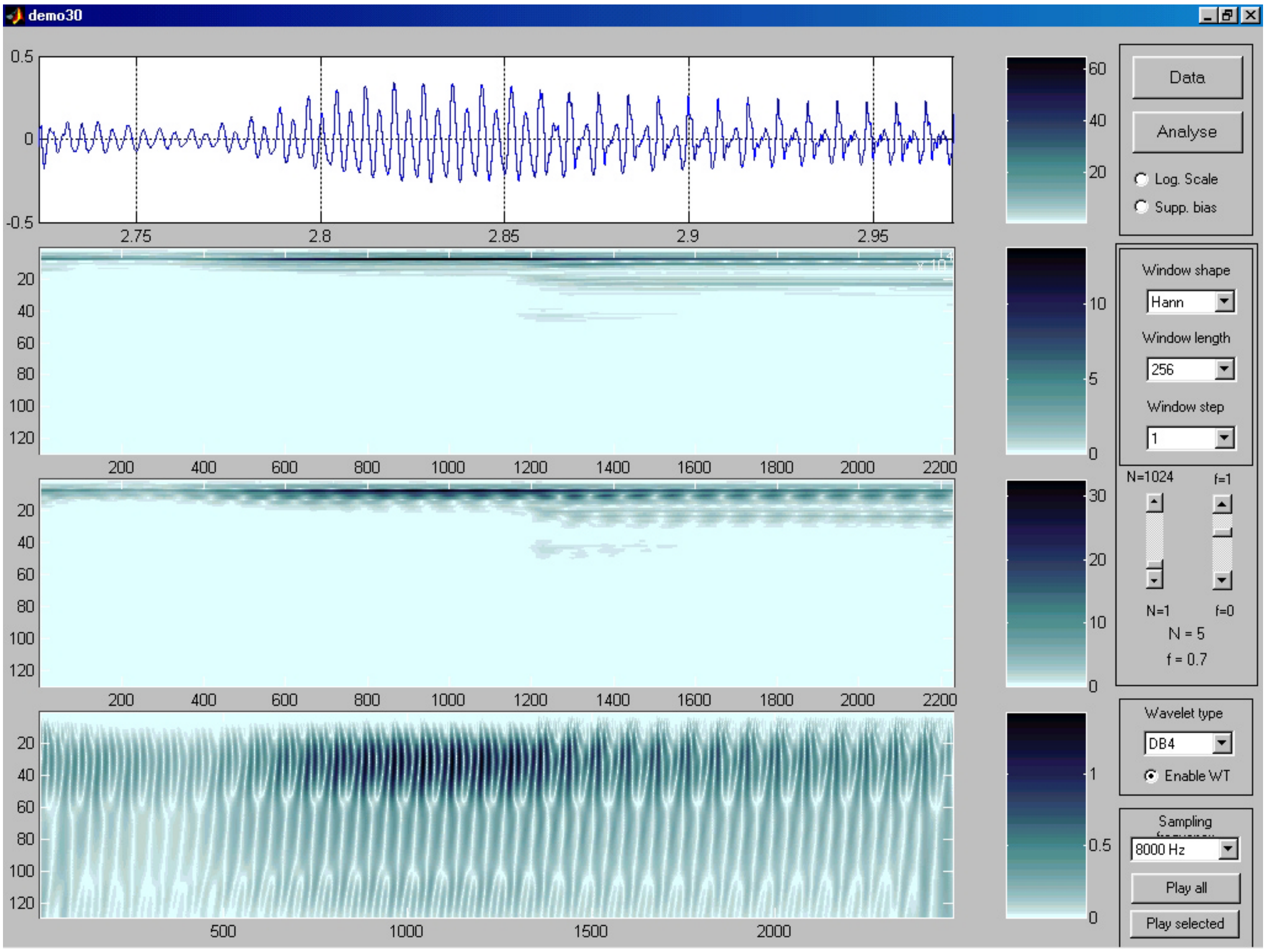

Figure 5: A piece of speech signal. First subsection shows time domain, the second one spectrum, Zolotarev transformed signal follows and finally the same signal transformed using Wavelet transform. Parameters of all methods are shown on the right hand side of the figure.

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