DESIGN OF CONTROL PROCESSES IN DPS BLOCKSET FOR MATLAB & SIMULINK

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Abstract

In the paper design of control processes for class of distributed parameter systems is presented. Controlled system is interpreted as lumped-input/distributed-output systems with dynamics modelled by finite element method. Schemes of distributed parameter control loops are arranged in software environment Distributed Parameter Systems Blockset for MATLAB & Simulink, which is third-party MathWorks product. In conclusion, results of control of a melting glass temperature field are shown.

1   Introduction

Nowadays is paid considerable attention to the numerical dynamical analysis of diverse processes, machines, apparatuses, as dynamical systems given over complex shape of definition domains. New software systems based on <u>f</u>inite <u>e</u>lement <u>m</u>ethod (FEM) have been developed to treat these problems. In this sphere of applications emerges a new discipline – computational science and engineering. The fields of quantities to be studied are often monitored by camera methods or appropriate sensor fields. In accordance with systems and control theory, all these real systems are <u>d</u>istributed <u>p</u>arameter <u>s</u>ystems (DPS). Their state or output variables are fields of variables, or spatial distributed variables. In practice, controlled DPS frequently are found as <u>l</u>umped-<u>i</u>nput/<u>d</u>istributed-output <u>s</u>ystems (LDS).

In the paper first basic concept of LDS dynamics and control will be outlined. Further basic features of the DPS Blockset and procedure of control synthesis design of glass melting temperature field with dynamics given by FEM will be described. In conclusion, results of control of a melting glass temperature field as DPS/LDS are shown.

2   Basic concept of DPS/LDS dynamics and control

In general, DPS are systems whose state or output variables, X(x,y,z,t)/Y(x,y,z,t) are distributed variables or fields of variables, where (x,y,z) is a vector in 3D. In the mathematical theory DPS are interpreted as systems whose dynamics is described by <u>p</u>artial <u>d</u>ifferential <u>e</u>quations (PDE), Lions (1971). In the input-output relation, PDE define <u>d</u>istributed-input/<u>d</u>istributed-output <u>s</u>ystems (DDS) between distributed input U(x,y,z,t) and distributed output variables Y(x,y,z,t), at initial and boundary conditions given, Fig. 1.

Figure 1: Distributed-input/ distributed-output system

It is well-known, that formulation and solution of control tasks for real DPS based on DDS is too difficult. Much better representation of DPS is in the form of LDS, (Hulkó,1987, 1998) see Fig. 2.

Figure 2: Lumped-input/distributed-output system: $\{U_i(t)\}_{i=1,n}$ - lumped input variables, Y(x,y,z,t) - distributed output variable

Dynamics of LDS is decomposed to time and space components. In the time dependency, there are for example discrete transfer functions (1) between i-th input variable and corresponding partial distributed output variable at point $\overline{x}_i = (x_i, y, z)$ for i=1,n.

$$\left\{ SH_i\left(\overline{x}_i, z\right) \right\}_{i=1,n} \tag{1}$$

In the space dependency there are e. g. reduced transient step responses in steady-state:

$$\left\{ \mathrm{H}HR_i\left(\overline{x}, \infty\right) \right\}_{i=1,n} \tag{2}$$

Decomposition of dynamics enables also to decompose the control synthesis to time and space control tasks in distributed parameter control loop, see Fig. 3.
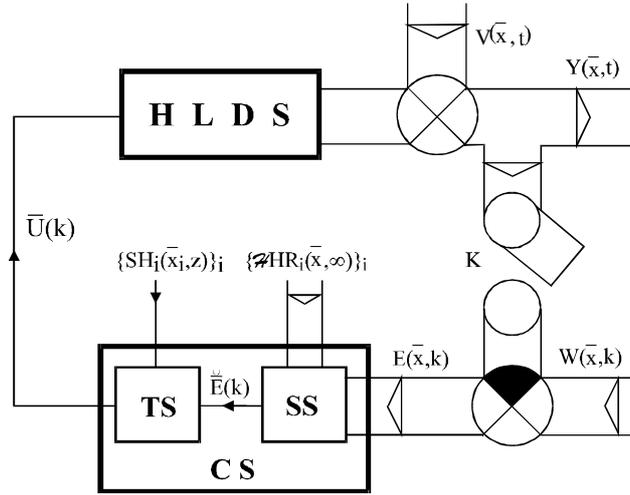

Figure 3: Distributed parameter discrete feedback control loop at unit sampling period
HLDS - LDS with zero-order holds $\{H_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, K - time/space sampling, $\mathrm{Y}(\overline{x}, t)$ - distributed controlled variable, $\mathrm{W}(\overline{x}, k)$ - control variable, $\mathrm{V}(\overline{x}, t)$ - disturbance variable, $\mathrm{E}(\overline{x}, k)$ - control error

Let the goal of control be to ensure the steady-state control error to be minimal, i.e.:

$$\min \left\| E\left(x, \infty\right) \right\| = \min \left\| W\left(x, \infty\right) - Y\left(x, \infty\right) \right\| = \left\| W\left(x, \infty\right) - \breve{Y}\left(x, \infty\right) \right\| = \left\| \breve{E}\left(x, \infty\right) \right\| \tag{3}$$

where $\left\| . \right\|$ is a norm appropriately chosen.

First, the approximation problem (4) in the block of the <u>S</u>pace <u>S</u>ynthesis (SS) is solved on the set of reduced steady-state distributed step responses $\left\{ \mathrm{H}HR_i\left(x, \infty\right) \right\}_{i=1,n}$,

$$\min_{E_i} \left\| E\left(x, k\right) - \sum_{i=1}^{n} E_i\left(k\right) \mathrm{H}HR_i\left(x, \infty\right) \right\| = \left\| E\left(x, k\right) - \sum_{i=1}^{n} \breve{E}_i\left(k\right) \mathrm{H}HR_i\left(x, \infty\right) \right\| \tag{4}$$

Further, the control errors vector $\breve{\bar{E}}\left(k\right) = \left\{ \breve{E}_i\left(k\right) \right\}_i$ enters the <u>T</u>ime <u>S</u>ynthesis block (TS), where, vector components $\left\{ \breve{E}_i\left(k\right) \right\}_i$ are fed through the inputs of controllers $\left\{ R_i\left(z\right) \right\}_i$ and the sequence of control variables $\overline{U}\left(k\right)$ is generated. Tuning of controllers is done - according to single components of the time part of the controlled distributed parameter system dynamics $\left\{ SH_i\left(x_i, z\right) \right\}_{i=1,n}$.

During the control process, for $k \rightarrow \infty$, we get the following relation and thus, the control task (3) is accomplished.

$$\min_{E_i} \left\| E\left(x, \infty\right) - \sum_{i=1}^{n} E_i\left(\infty\right) \mathrm{H}HR_i\left(x, \infty\right) \right\| = \left\| \breve{E}\left(x, \infty\right) \right\| \tag{5}$$

3 Modeling of temperature field of glass melting furnace

Melting glass process is typical case of DPS. In the input/output relation it is possible to model it as LDS. Lumped inputs set flow rates of heating medium (earth gas and air) into series of burners located on both sides of the glass furnace above molten glass. Temperature field of the molten glass on the definition domain $\Omega \in E_2$ (in cross-section of the melting space) is distributed output variable, see Fig.4.

Dynamics of a melting glass process as DPS is modelled in FEMLAB (COMSOL Multiphysics) software environment, which offers very efficient tool for modelling and simulation scientific and engineering problems based on PDE. Solution of formulated models is realised by FEM.

Is well know, that FEM is a method for solving an equation by approximating continuous quantities as a set of quantities at discrete points, often regularly spaced into a so-called grid or mesh. Because finite element methods can be adapted to problems of great complexity and unusual geometry, they are an extremely powerful tool in the solution of important problems in heat transfer, fluid mechanics, and mechanical systems. Furthermore, the availability of fast and inexpensive computers allows problems which are intractable.

For the recuperative gas glass tank furnace with the cross flame was made two-dimensional space model of the melting glass dynamics based on FEM. In the definition domain is supposed glass melting in steady-state and heating by the mixture of gas and air, which enter into four pairs of burners. Distribution of temperatures in molten glass in the definition domain $\Omega \in E_2$ is modelled by PDE of parabolic type

$$d\frac{\partial Y}{\partial t} - \nabla(c\nabla Y) + aY = f \tag{6}$$

with constants d, c, a and Neumann type boundary conditions

$$\vec{n}(c\nabla Y) + qY = g \tag{7}$$

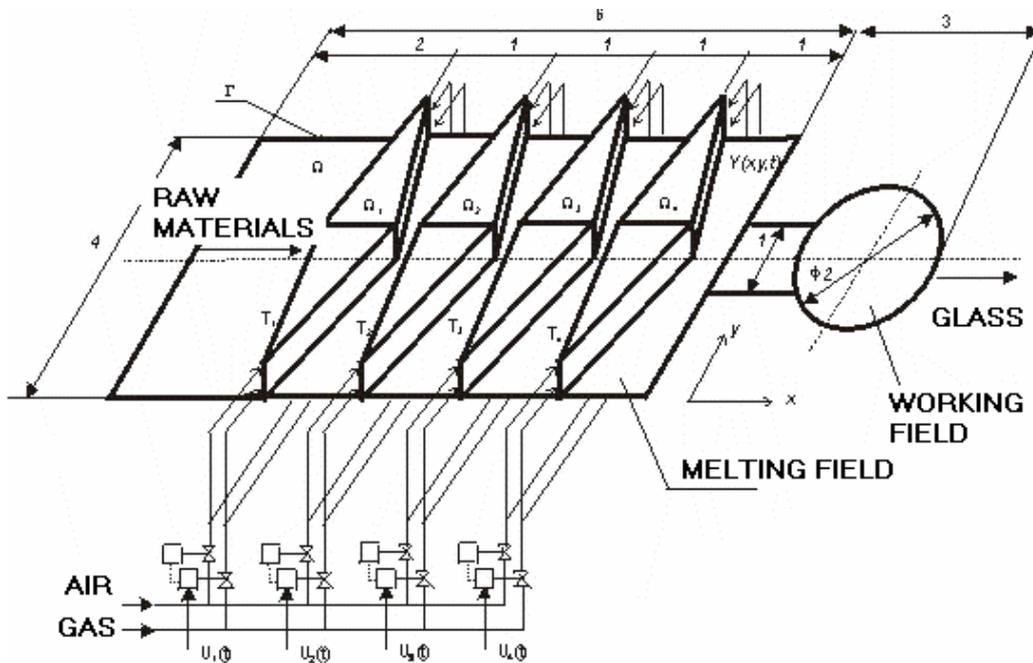where $\vec{n}$ is the outward unit normal and q = 0, g = 0.

Figure 4: Scheme of recuperative gas glass tank furnace

Modeling process in Femlab is started via Model Navigator, which enables to select Application Modes. In this case Heat Transfer and Space dimension – 2D was chosen and GUI was activated. There through the toolbar menu both definition domain $\Omega \in E_2$ and subdomains were drawn, see Fig. 5, further in Physics menu parameters of equation system were defined. For this model distributed transient responses from each input actuated on subdomains were obtained, see Fig. 6.
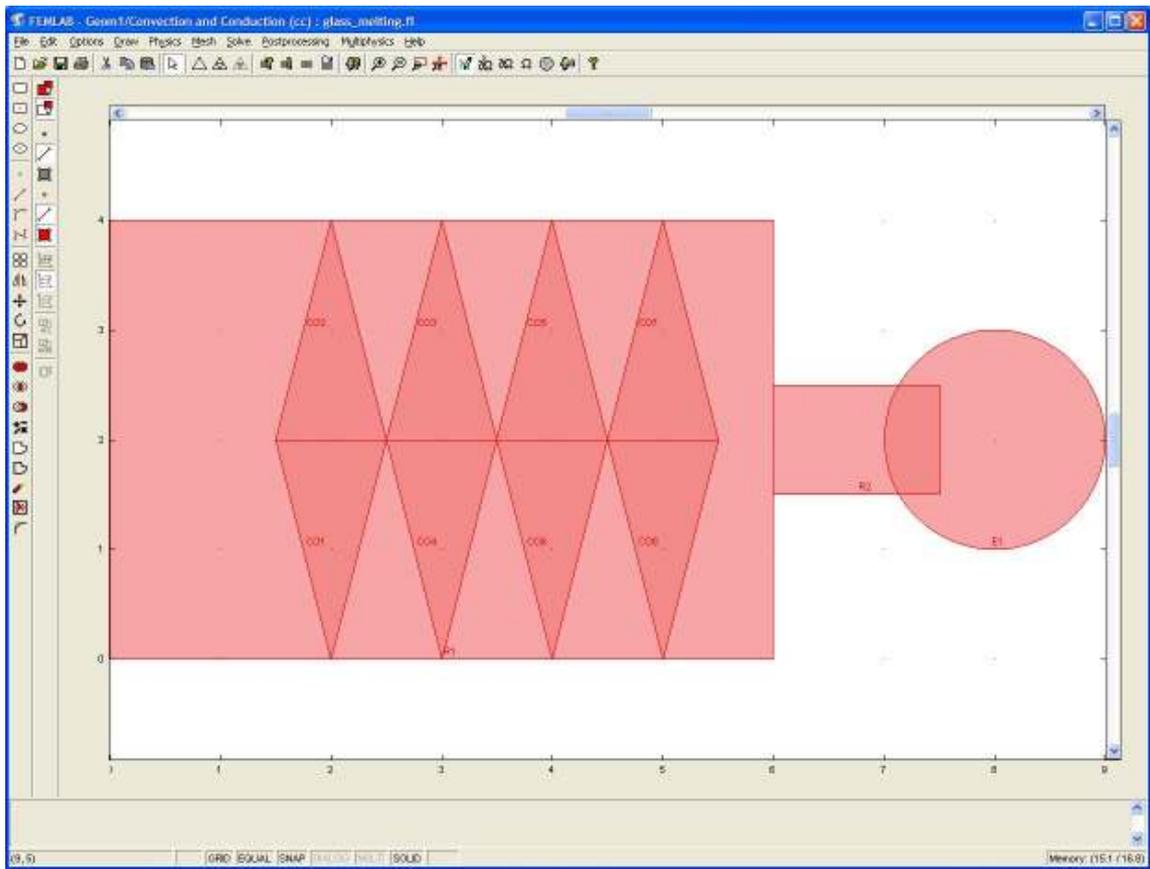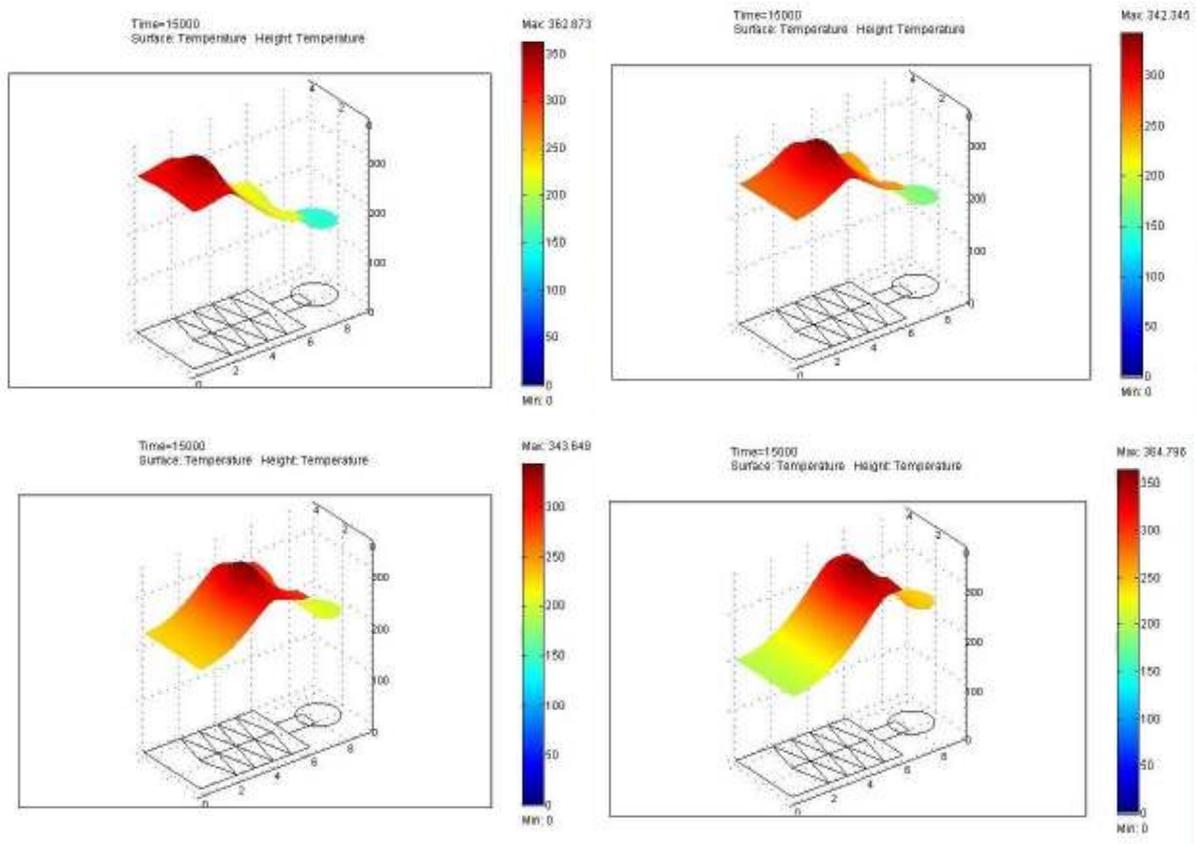
Figure 5: Definition domain

Figure 6: Distributed transient response in steady-state from each lumped input

On distributed transient step responses in steady-state, point with maximal value was determined. Partial distributed transient response in this point was identified, see Fig.7 and discrete transfer function (1) was obtained for i=1, 4 inputs. These transfer function are used for time synthesis. Reduced transient step responses in steady-state (2) for space synthesis have been also expressed, see Fig. 8.

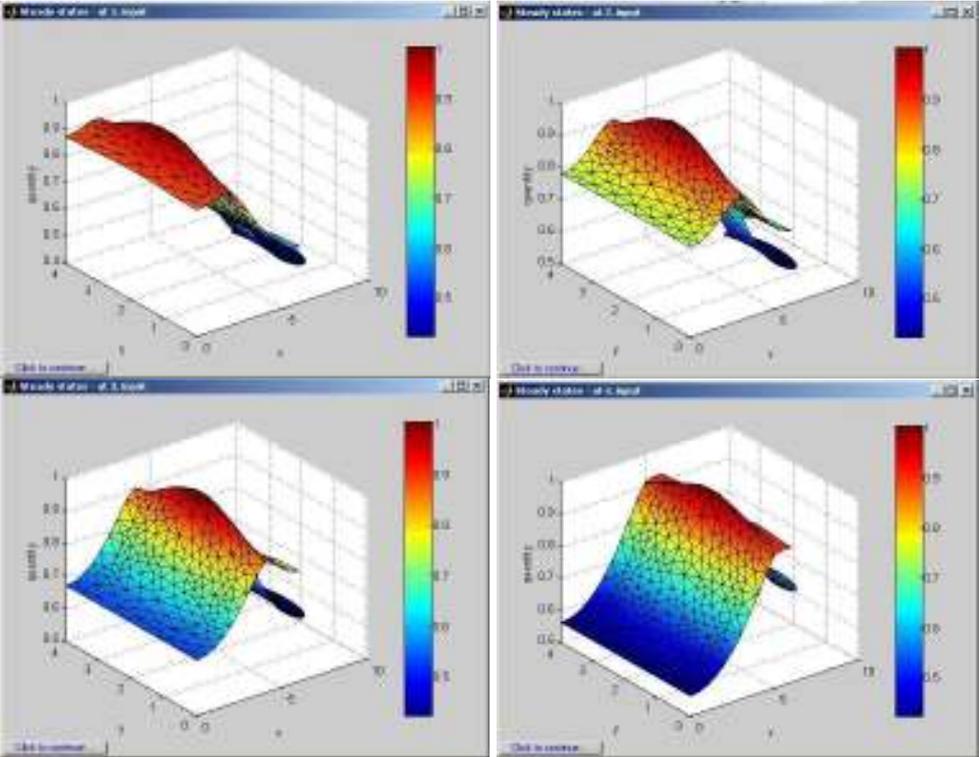Figure 7: Identification of partial distributed transient responses from each lumped input

Figure 8: Reduced distributed transient responses in steady-state from each input

4 Distributed Parameter Systems Blockset for MATLAB & Simulink

The Distributed Parameter Systems Blockset is a blockset for use with MATLAB & Simulink for distributed parameter systems and their applications in modeling, control and design of dynamical systems given on complex 3D domains of definition, see Fig. 9. – www.dpscontrol.sk .
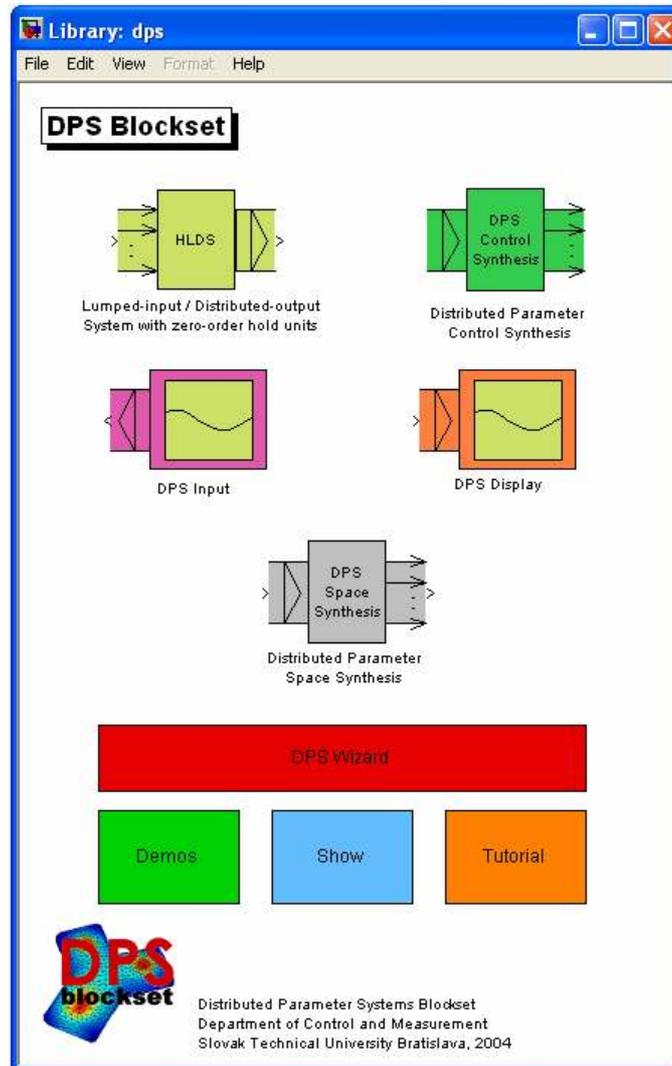

Figure 9: DPS Blockset for MATLAB & Simulink

The blockset features:
- Engineering methods for DPS modeling, control and design
- DPS models based on lumped-input/distributed-output systems, time/space analysis, synthesis and design tools
- Distributed parameter PID, algebraic, state space and robust control schemes
- DPS Wizard demonstrates in step-by-step operation distributed parameter control loops arrangement and setting procedures
- Suite of blocks and schemes for DPS control practically in any field of technical practice
- Interactive Control Service for support DPS control solutions via the internet

The block **HLDS** models controlled distributed parameter systems as lumped-input/distributed-output systems with zero-order hold units. The **DPS Control Synthesis** provides feedback to distributed parameter controlled systems in control loops with blocks for **PID, algebraic, state space and robust** control. The block **DPS Input** generates distributed quantities which can be used as distributed control variables or distributed disturbances, etc. **DPS Display** presents distributed quantities with many options including export to AVI files. The block **DPS Space Synthesis** performs space synthesis as an approximation problem. The block **Tutorial** presents methodological framework for formulation and solution of distributed parameter systems of control. The block **Show** contains

motivation examples: Control of temperature field of 3D metal body, Control of 3D beam of „smart"
structure, Adaptive control of glass furnace and Groundwater remediation control. The block **Demos**
contains examples oriented to methodology of modeling and control synthesis. The **DPS Wizard** in
step-by-step operation, by means of five model examples on 1D-3D with default parameters, gives a
guide for arrangement and setting distributed parameter control loops.

5   Simulation of control process

In the MATLAB & Simulink environment by means of the DPS Blockset distributed parameter
system of control is arranged, see Fig. 10. It is feedback control loop, where block Distributed
Parameter control Synthesis includes both time part and space part of control synthesis. In this case
control system consist of four single parameter control loops, where discrete PID controllers are used.
For tuning of controllers, discrete transfer functions (1) are at disposal.

Simulation results of control process of temperature field of melting glass are on Figures 11, 12.
Control objective is certain temperature required by technology with 10% divergence at most. Glass
melting process is long-term process with slow dynamics. It is energy demanding process. Therefore,
time of steady state acquirement is not as crucial as fuel consumption. With proper set up of
controllers can be achieved considerable cost saving. Here controllers were tuned in order to assure
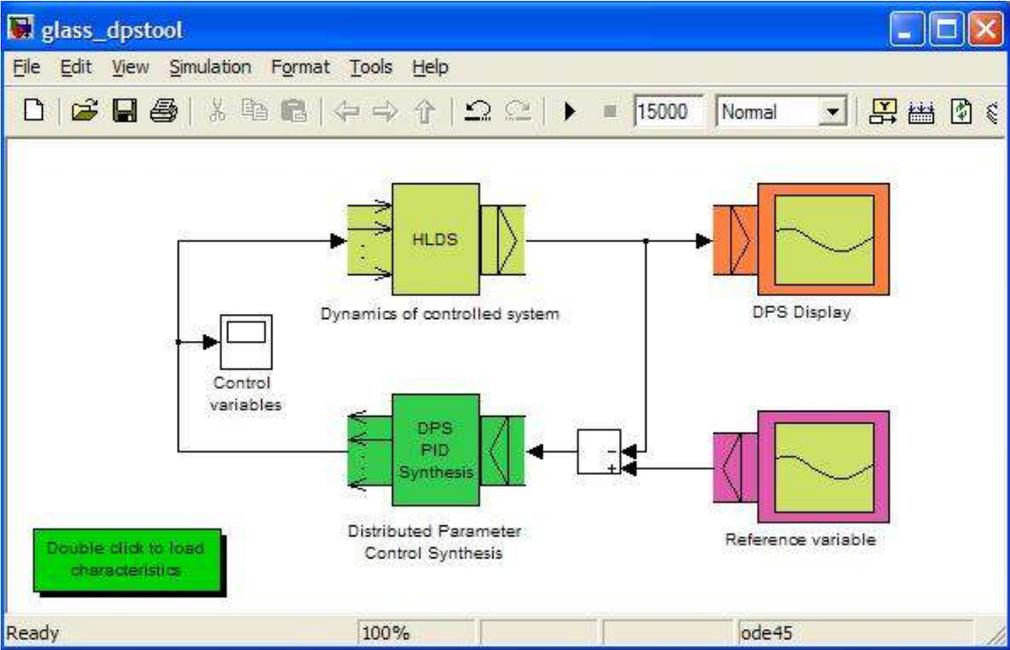aperiodic running of quadratic norm of distributed control error.

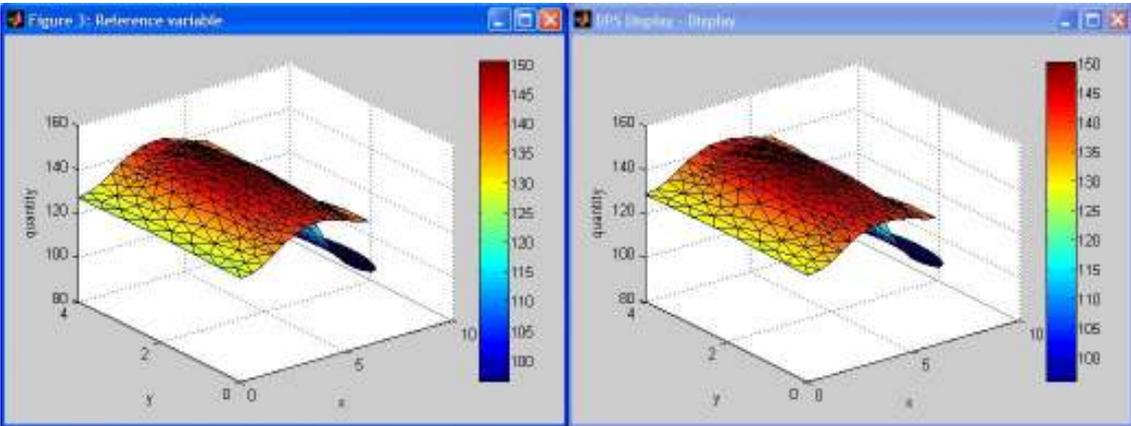Figure 10: DPS control loop in DPS Blockset for MATLAB & Simulink

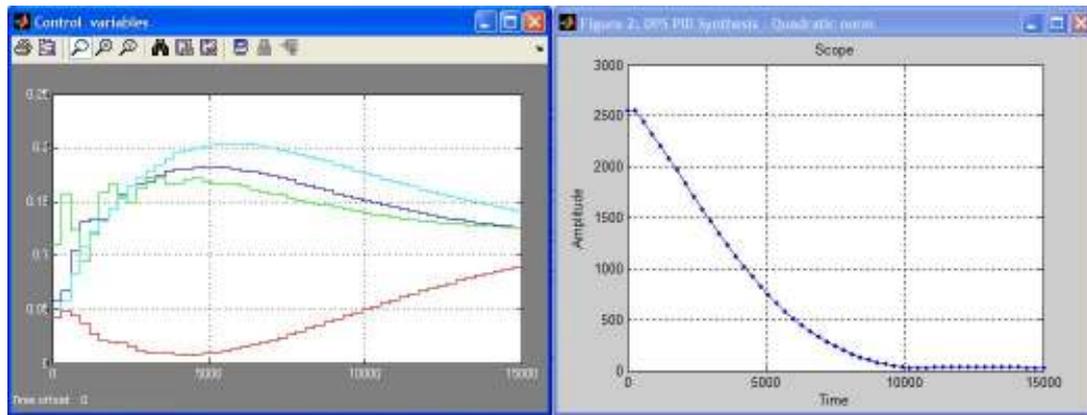Figure 11: Distributed reference variable W(x,y,t) and controlled variable Y(x,y,t), t→∞

Figure 12: Control variables $U_i(k)$ and quadratic norm of distributed control error $\|E(k)\|$

6   Conclusion

In today's engineering environment huge amount of money are invested into development. Devices have to operate more precisely and react more robustly than ever before. FEM and FEA (Finite Element Analysis) has shown us its strong points. Moreover, together with computer technologies headway it is taking valid position in engineering field.

Methodical approach presented in the paper demonstrates simple possibilities, how to exploit of distributed dynamical characteristics, obtained by numerical analysis of systems on complex definition domains for control of distributed parameter systems. The DPS Blockset for MATLAB & Simulink provides block-oriented efficient software for this kind of tasks.

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