PLANAR ANISOTROPY OF FIBRE SYSTEMS BY USING MATLAB IMAGE PROCESSING TOOLBOX

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Abstract: This paper describes a simple method of description of fibre systems anisotropy using image analysis. This method is based on Fourier transform which is in frequency domain displayed by high values of frequency components corresponding with gradient of image function in spatial domain. The values of frequency components are added for directional vectors depending on certain angle and brought up to polar diagram and histogram. The polar diagram can be seen as an estimate of the rose of directions.

Key words: Anisotropy; Fourier transform; Rose of directions

Introduction

The article is aimed at graphical description of planar anisotropy of fibre or other planar systems based on image analysis. The method used spectral techniques with the aid of two-dimensional Fourier transform. The parts of image understood as objects representing entities of our world are concerned according to further processing. These objects are either randomly placed or they prefer certain directional placement. The objects should be in contrast with the background (gradient of image function on the edges of object and background). In textile experience, the objects are considered to be fibres, threads, cross – sections of fibres etc., systems containing objects can be webs, fibre layers, woven fabrics, knitted fabrics, non-woven textiles etc.

Anisotropy of fibre systems

According to [1] the characteristic of planar anisotropy is angular density of length of thread or fibres $f(\alpha)$, which defines the length of thread or fibres bounded to angular distance $\alpha \pm \alpha / 2$. Function $f(\alpha)$ or rather polar plot of density $f(\alpha)$ is called the rose of directions. An experimental graphical method for the estimation of $f(\alpha)$ is described in [1, 2]. This method uses the net of angles $\alpha_1 \dots \alpha_n$ situated on the top of monitored fibre system for the construction of rose of intersections. Rose of direction as an estimation of function $f(\alpha)$ is then obtained from the rose of intersections through the graphical construction of Steiner compact. The limit number of angles is $n \leq 18$.

Proposed graphical method is based on the method of image analysis with the spectral approach. The goal of this method is fast graphical representation of directional arrangement of objects (estimation of anisotropy $f(\alpha)$) in the form of rose of directions and histogram.

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2D Fourier Transform (2DFT)

Spectral approach is based on two-dimensional (2D) Fourier transform (FT) and is suitable for describing the textured images. The dominating directions (main directions of brightness dividing lines or gradient of image function) in the directional textures (spatial domain) correspond to large magnitude of frequency components distributed along the straight lines in the Fourier spectrum (frequency domain). In contrast, the random nature of statistical textures causes that the frequency components in the power spectrum are approximately isotropic and possess nearly circular shape. The Fourier transform is rotation dependent, i.e. rotating the original image by an angle will rotate its corresponding frequency plane by the same angle. The transform of horizontal lines in spatial domain image appears as vertical lines in the Fourier domain image, i.e. the lines in the spatial domain image and its transformation are orthogonal to each other [5]. Let $f(x,y)$ be the gray level at pixel coordinates (x,y) in the image of size M x N. For image the direct and inverse Fourier transform are given:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M+vy/N)} , \qquad (1)$$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M+vy/N)} , \qquad (2)$$

where u = 0, 1, 2 ... N - 1 and, v = 0, 1, 2 ... M - 1 are frequency variables [3]. If $f(x,y)$ is real, its transform in general is complex. $R(u,v)$ and $I(u,v)$ represent the real and imaginary components of $F(u,v)$, the Fourier spectrum is defined as:

$$|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)} \, . \qquad (3)$$

The power spectrum (called also the power spectral density) is defined by:

$$P(u,v) = |F(u,v)|^2 . \qquad (4)$$

To show the spectrum scaled to 8 - bit grey levels, $P(u,v)$ is converted [1, 2]:

$$P(u,v) = \log(1 + |F(u,v)|^2) . \qquad (5)$$

If $f(x,y)$ is real, its Fourier transform is conjugate symmetric about the origin, that is:

$$F(u,v) = F^*(-u,-v), \qquad (6)$$

which implies that the Fourier spectrum also is symmetric about the origin:

$$|F(u,v)| = |F(-u,-v)| . \qquad (7)$$

Fig. 1(a1) - (c1) represent binary images of simulated structural lines in the 0° direction, 45° direction, in the interval 30° - 60° respectively. The length, position and orientation of lines were randomly generated from uniform distribution. Fig. 1(a2) - (c2) shows Fourier frequency spectrum scaled into 256 grey levels. As can be seen from these figures, information about direction of major structural lines in the spatial domain is concentrated in the Fourier domain image as a direction of corresponding large magnitude frequency components (represent with white colour).

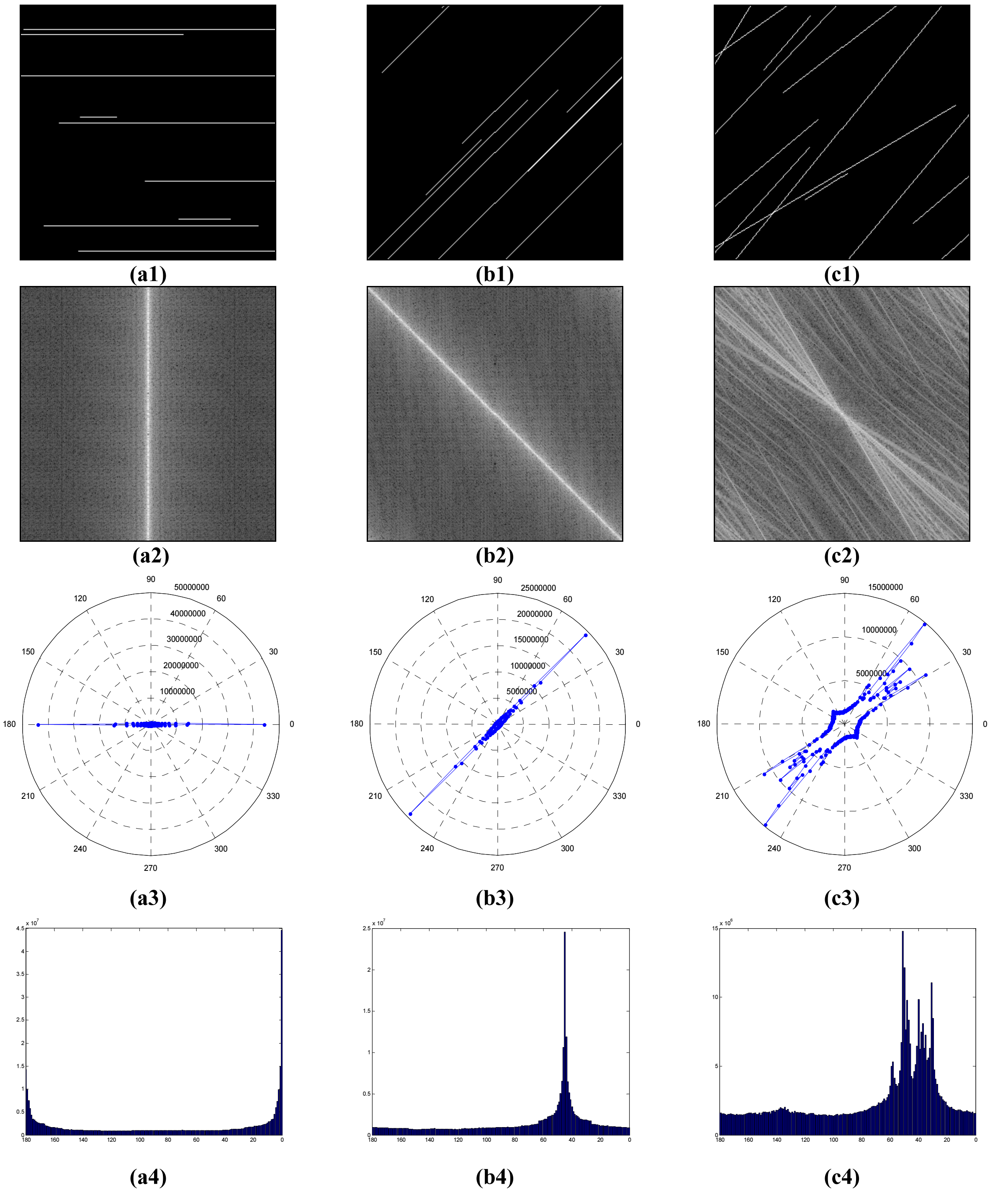


Fig. 1. *(a1) - (c1) Binary images of simulated structural lines, (a2) - (c2) Fourier frequency spectrum as an intensity image, (a3) - (c3) polar plot of* S_α*, (a4) - (c4) histogram of* S_α*.*

Assumptions

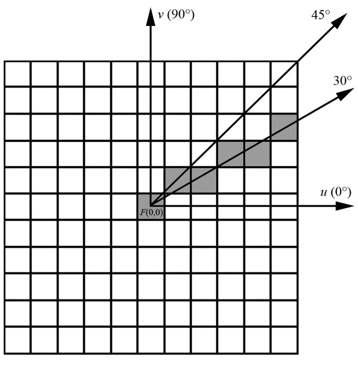
Image matrix is square that is of size *M x M*. Let *M* is odd number by reason of specification of Fourier spectrum origin and image matrix is scaled to 8 – bit grey levels (monochromatic image). All frequency components from the Fourier frequency spectrum are added together in the directional vector of certain angle α. Since the transform of real image function $f(x,y)$ is complex number, the absolute magnitudes of frequency components $|F(u,v)|$ are added according to relation (3). Sum of frequency components S_α in the directional vector is given:

$$S_\alpha = \sum_{i=1}^{(M+1)/2} \left|F(u,v)\right|. \qquad (8)$$

where α form an angle between the directional vector and abscissa (u axis, 0°), $|F(u,v)|$ is a frequency component of directional vector at the coordinates (u,v) and M is a size of an image.

Computation of directional vector coordinates

As can be seen from equation (7), Fourier frequency spectrum is symmetric about the origin; it is sufficient count up frequency components of directional vectors depending on α in the interval $(0,\pi)$, i.e. specify coordinates for the I. and II. quadrant. I. and II. quadrant coordinates are symmetric about the ordinate (v axis, $\pi/2$), that is $(u,v) = (-u,v)$, therefore sufficient condition is determination of coordinates for I. quadrant:



$$\begin{aligned} 0 \le \alpha \le \frac{\pi}{4} \quad &\rightarrow \quad v = u.\tan\alpha \\ \frac{\pi}{4} < \alpha \le \quad &\rightarrow \quad u = \frac{v}{\tan\alpha}, \end{aligned} \qquad (9)$$

Fig. 2. Coordinates for directional vector dependent on $\alpha = 30°$.

where u is abscissa axis or column number, v is ordinate or row number and coordinates (u,v) are rounded to closest integer, because the coordinates acquired integer discrete value. The DC (Direct Current) component is the origin of frequency domain $F(0,0)$, and represent the origin of system of coordinates. Fig. 2 displays example of coordinates for directional vector in I. quadrant, $\alpha = 30°$.

For an estimation of the rose of directions the magnitude of S_α is plotted into the polar diagram and consequently into the histogram. For realization of proposed method, the algorithm was created in MATLAB programming language (Image Processing Toolbox). Input parameters are image matrix and the output is the visualization of direction arrangement of objects in the form of polar plot of S_α (can be seen as estimation of rose of direction) and histogram of S_α.

Comparison with the rose of directions by means of Steiner compact

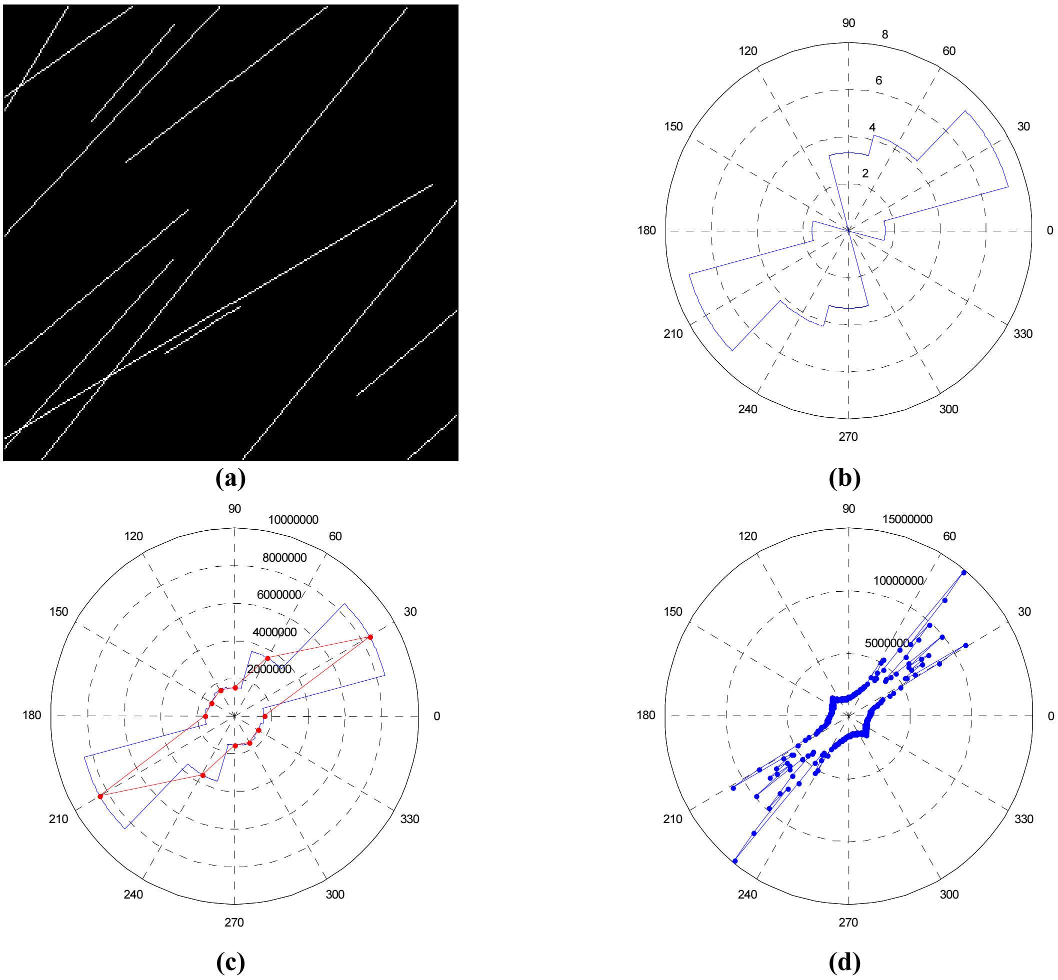


Fig. 3. *(a) Simulated fibre system, (b) estimation of the rose of directions by means of Steiner compact, (c) estimation of the rose of directions by using the Fourier transform, plot in 30 degrees step, (d) estimation of the rose of directions by using the Fourier transform, plot in 1 degree step.*

Fig. 3(a) display binary image of simulated structural lines from Fig. 1(c1) and corresponding estimation of the rose of direction achieved by means of Steiner compact in six directions $\alpha_k = k\pi/6$ for $k = 1,\ldots,5$. In Fig. 3(c) red line display estimation of the rose of direction also in six directions and (d) in directions in one-degree step by using image analysis with the aid of Fourier transform. Fig. 1(a3) - (c3) display polar plot of S_α and represent the estimation of function $f(\alpha)$ (rose of directions), and 1(a5) - (c5) display histogram of S_α for the binary images from the Fig. 1(a1) - (c1).

Fig. 4(a1) is the grey level image of random Gaussian noise as the isotropic system. It can be seen from the polar plot and histogram of S_α on Fig. 4(a3),(a4), the magnitudes of S_α are uniformly distributed along the whole spectrum of angles and system is not anisotropy.

Similarly, Fig. 4(b1),(c1) show the grey level images of nanofibres captured by the screening electron microscope VEGA-TESCAN with the randomly distributed structure, Fig. 4(b2),(c2) represent Fourier frequency spectrum, 4(b3),(c3) polar plot of S_α and 4(b4),(c4) histogram of S_α. As can be seen from polar plots, the structure of image of nanofibres in Fig. 4(b) is almost isotropic, but structure in Fig. 4(c) show preference of directional placement of fibres in 90°-120° direction.

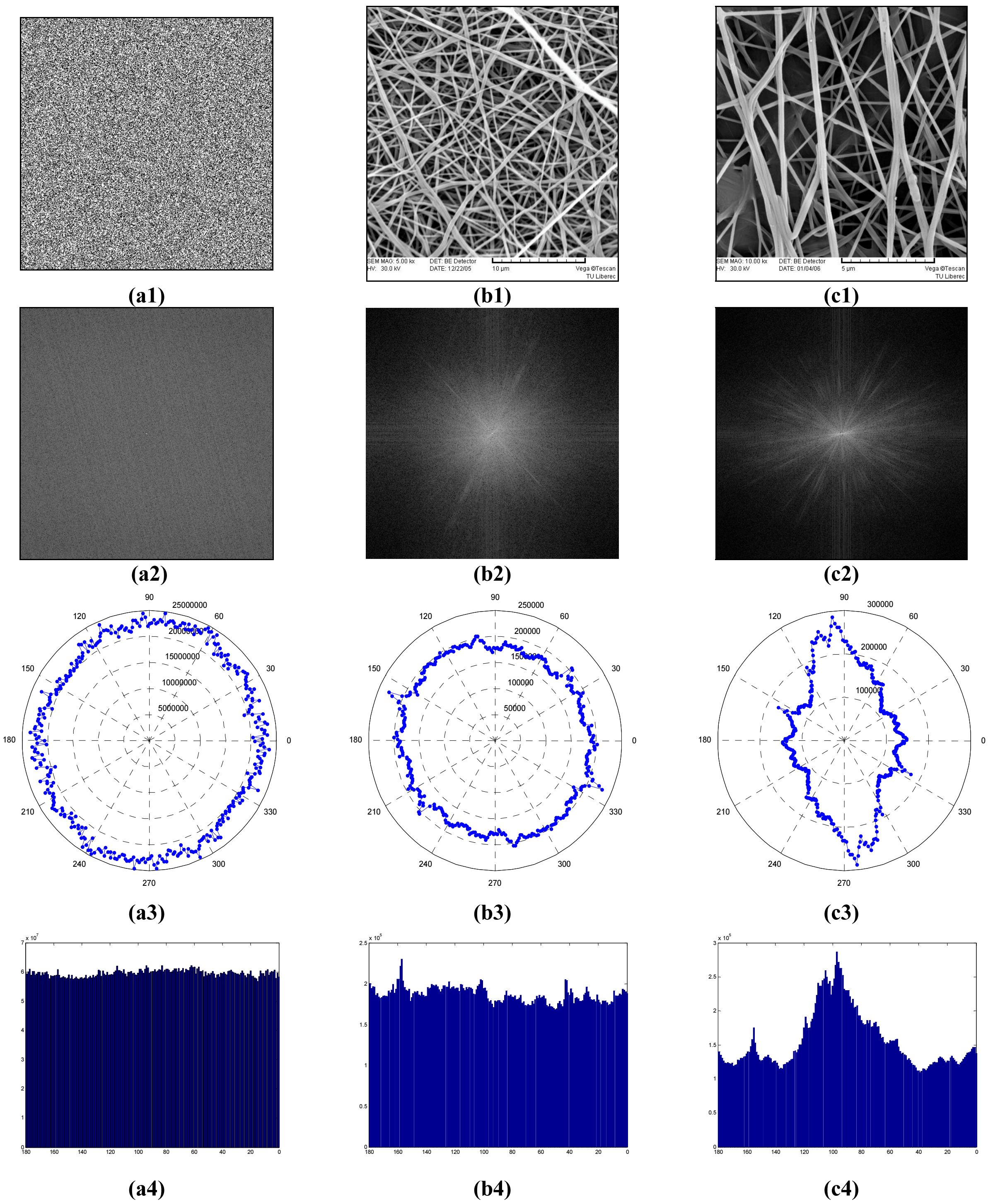


Fig. 4. *(a1) - (c1) Textured images, (a2) - (c2) Fourier frequency spectrum as an intensity image, (a3) - (c3) polar plot of S_α, (a4) - (c4) histogram of S_α.*

Matlab Web Server application

As mentioned above the m-file was made and consequently modified for creation of dynamic WWW application with the aid of Matlab Web Server. The data are sending through the WWW to Matlab Web Server for computing and the results are displaying by the web browser. The main purpose of realization of those applications is enhancement of educational system in Technical university of Liberec. Fig. 5 displays dynamic application, which evaluates anisotropy of fibre or other system. Input parameter is image matrix and the output is the estimation of anisotropy in the form of polar plot and histogram.

The link for application can be found on address
http://e-learning.tul.cz/cgi-bin/elearning/elearning.fcgi?ID_tema=73&stranka=publ_tema

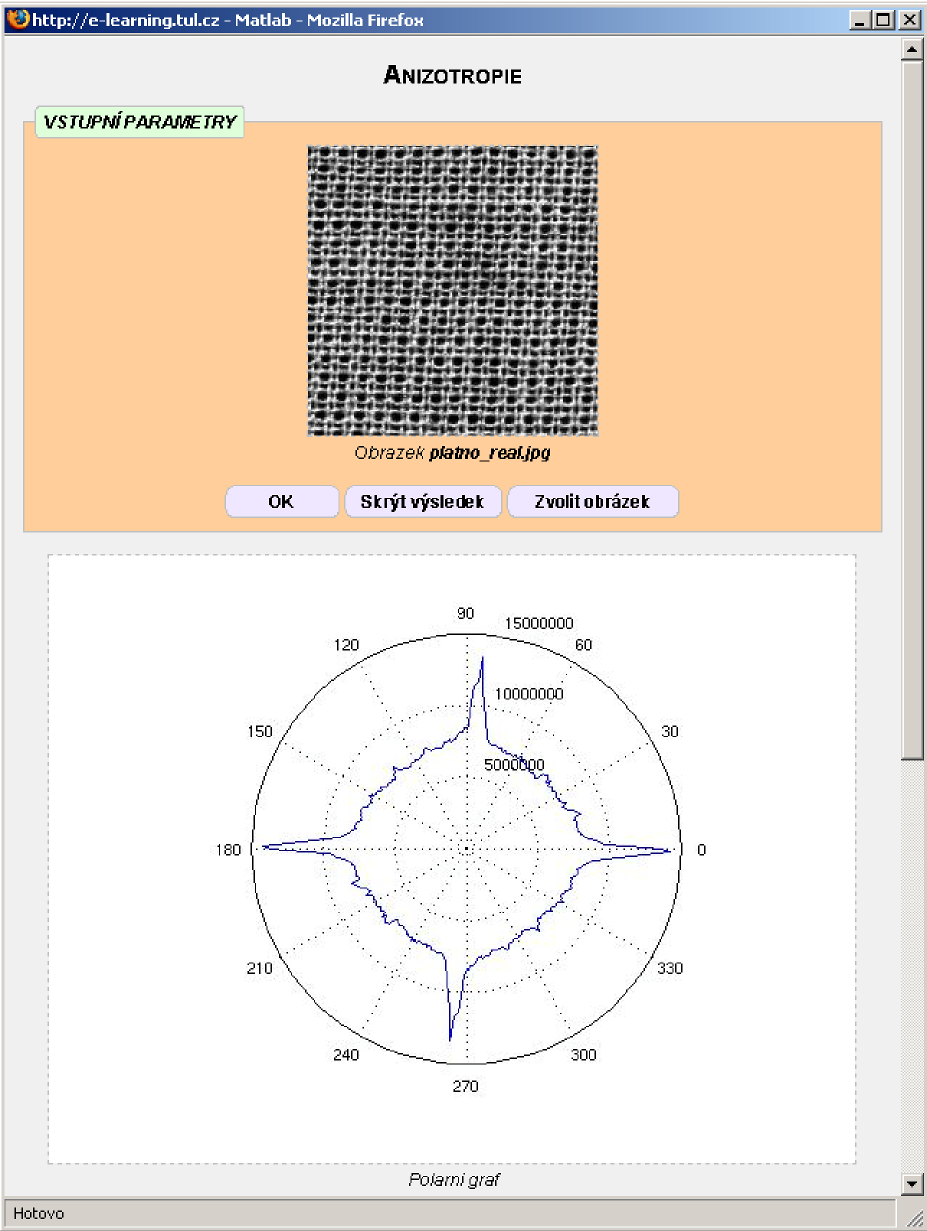


Fig. 5. Anisotropy of fibre systems as dynamic WWW application.

Conclusion

This paper presents simple graphical method of planar anisotropy of fibre systems. Advantage of this method is the fastness; result is directly available after the acquisition of image and application of algorithms. Anisotropy visualization is in the form of polar diagram and histogram. Polar diagram can be seen as an estimation of the rose of directions or function $f(\alpha)$. Is it possible to monitor direction vectors with the angular step 1°. Method can be used for anisotropy of other systems too.

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