SIMULATION OF ROBUST CONTROL OF TEMPERATURE FIELDS IN DPS BLOCKSET FOR MATLAB & SIMULINK

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Abstract

In the paper simulation of a robust control of temperature fields of the melting glass is presented. Controlled system is interpreted as lumped-input/distributed-output systems with dynamics modelled and studied by a finite element method in the COMSOL Multiphysics software environment. For the robust control synthesis with an internal model control structure, a model uncertainty of the controlled system was considered and robust control loops were arranged. Simulation was realized using the Distributed Parameter Systems Blockset for MATLAB & Simulink, which is third-party MathWorks product.

1 Introduction

Nowadays a number of software products based on finite element method (FEM) are at disposal for numerical dynamical analysis of machines and processes practically in all engineering disciplines. Most of the analyzed dynamical systems in fact are distributed parameter systems (DPS) given by numerical structures on complex-shape 3D definition domains. There is possible to interpret DPS as lumped-input/distributed-output systems (LDS). Dynamical characteristics obtained by these numerical methods offer wide possibilities for control of systems as DPS.

In the paper first basic concept of LDS dynamics and robust control is outlined. Dynamics of a melting glass process as DPS is modelled in the COMSOL Multiphysics software environment. It is very efficient tool for modeling and simulation scientific and engineering problems based on partial differential equations (PDE). Solution of formulated models is realized by FEM and numerical models of melting glass temperature fields are obtained in the LDS form. Based on uncertainty analysis of the models, a robust control synthesis for internal model control (IMC) is applied. Simulation of the robust control of temperature fields is realized using the Distributed Parameter Systems Blockset for MATLAB & Simulink, a third-party software product of The MathWorks, Inc., developed at the Institute of automation, measurement and applied informatics, Faculty of Mechanical Engineering STU (Hulkó 2004).

2 Basic concept of DPS/LDS dynamics and control

In general, DPS are systems whose state or output variables, X(x,y,z,t)/Y(x,y,z,t) are distributed variables or fields of variables, where (x,y,z) is a vector in 3D. In the mathematical theory DPS are interpreted as systems whose dynamics is described by partial differential equations (PDE), Lions (1971). In the input-output relation, PDE define distributed-input/distributed-output systems (DDS) between distributed input U(x,y,z,t) and distributed output variables Y(x,y,z,t), at initial and boundary conditions given. It is well-know, that formulation and solution of control tasks for real DPS based on DDS is too difficult. Much better representation of DPS is in the form of LDS, (Hulkó, 1987, 1998) see Fig. 1.

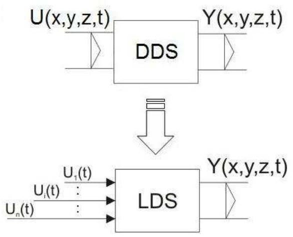

Figure 1: Representation of DPS is in the form of LDS:
$\{U_i(t)\}_{i=1,n}$ - lumped input variables, Y(x,y,z,t) - distributed output variable

Dynamics of LDS is decomposed to time and space components. In the time dependency, there are for example discrete transfer functions (1) between i-th input variable and corresponding partial distributed output variable at point $\overline{x}_i = (x_i, y_i, z_i)$ for i=1,n.

$$\left\{SH_i\left(\overline{x}_i, z\right)\right\}_{i=1,n} \tag{1}$$

In the space dependency there are e. g. reduced transient step responses in steady-state:

$$\left\{\mathcal{H}HR_i\left(\overline{x}, \infty\right)\right\}_{i=1,n} \tag{2}$$

Decomposition of dynamics enables also to decompose the control synthesis to time and space control tasks in distributed parameter control loop, see Fig. 2, where the goal of control is to ensure the steady-state control error to be minimal:

$$\min\left\|\overline{E}\left(x,\infty\right)\right\| = \min\left\|W\left(\overline{x},\infty\right) - Y\left(\overline{x},\infty\right)\right\| \tag{3}$$

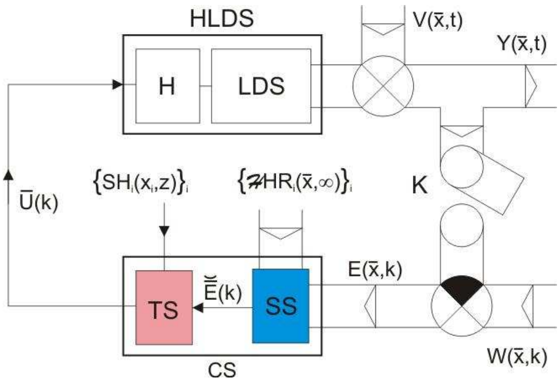

Figure 2: DPS feedback control loop:
HLDS - LDS with zero-order holds $\{H_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, K - time/space sampling, $Y(\overline{x}, t)$ - distributed controlled variable, $W(\overline{x}, k)$ - control variable, $V(\overline{x}, t)$ - disturbance variable, $E(\overline{x}, k)$ - control error

In the block SS, approximation of distributed control error $E\left(\overline{x},k\right)$, on the set of reduced steady-state distributed step responses $\left\{\mathcal{H}HR_i\left(\overline{x},\infty\right)\right\}_i$, is solved.

$$\min_{E_i}\left\|E\left(\overline{x},k\right) - \sum_{i=1}^{n} E_i\left(k\right)\mathcal{H}HR_i\left(\overline{x},\infty\right)\right\| = \left\|E\left(\overline{x},k\right) - \sum_{i=1}^{n} \breve{E}_i\left(k\right)\mathcal{H}HR_i\left(\overline{x},\infty\right)\right\| \tag{4}$$

Further, the control errors vector $\breve{\overline{E}}\left(k\right) = \left\{\breve{E}_i\left(k\right)\right\}_i$ enters into the block TS, where vector components $\left\{\breve{E}_i\left(k\right)\right\}_i$ are fed through the inputs of controllers $\left\{R_i\left(z\right)\right\}_i$ and the sequence of control variables $\overline{U}\left(k\right)$ is generated. Tuning of controllers is done - according to single components of the time part of the controlled distributed parameter system dynamics $\left\{SH_i\left(x_i, z\right)\right\}_{i=1,n}$.

3 Robust control system

In general a mathematical model for the plant dynamics is the basis for analysis and design of control systems. Also for LDS representation of DPS lumped and distributed models are used. However, in practice no mathematical model capable of exactly describing a physical process exists. It is obvious that although no model is able to represent the process perfectly, some of them will do so with greater accuracy than others.

The theory of robust control represents one of the possible approaches to the control system design in the presence of uncertainty. The goal of robust system design is to retain assurance of system performance in spite of model inaccuracies and changes. For the design techniques the following are supposed: formulation of nominal plant model, different plant uncertainty models and requirements both for stability and performance control.

3.1 Sources of uncertainties in the LDS structure and their description

LDS representation of DPS means decomposition of dynamics to space and time components. Both in time and space components uncertainties occur, therefore is very useful to regard it.

In distributed parameter control system according Fig. 2, single-input, single-output control loops in the block TS are tuned as closed feedback control loops using usual methods. In these loops, as models of the controlled system, transfer functions $\{SH_i(\bar{x}_i, z)\}_i$ eventually $\{S_i(\bar{x}_i, s)\}_i$ in the s-domain are used. These transfer functions describe the dynamics between sequences $\{U_i(k)\}_i$ and $\{Y_i(\bar{x}_i, k)\}_i$.

Sources of uncertainties are given by:

- procedure of modeling dynamics and possible change of parameters in models (1)
- solution of approximation problem (4), where lumped variables are obtained

In order to consider uncertainties, in this paper will be assumed that the dynamic behaviour of a plant is described not by a single linear time invariant model, but by a family $\mathbf{\Psi}_i$ of linear time invariant models. This family $\mathbf{\Psi}_i$ in the frequency domain is in following form:

$$\Psi_i = \left\{ S_i : \left| S_i(\bar{x}_i, j\omega) - S_i'(\bar{x}_i, j\omega) \right| \le \bar{L}_{ai}(\omega) \right\} \tag{5}$$

Here $S_i'(\bar{x}_i, j\omega)$ is the nominal plant. Any member of the family $\mathbf{\Psi}_i$ fulfils the conditions:

$$S_i(\bar{x}_i, j\omega) = S_i'(\bar{x}_i, j\omega) + L_{ai}(j\omega) \tag{6}$$

$$\left| L_{ai}(j\omega) \right| \le \bar{L}_{ai}(\omega) \quad , \forall \; S_i \in \Psi_i \tag{7}$$

where $L_{ai}(j\omega)$ is an additive uncertainty and $\bar{L}_{ai}(\omega)$ states a bound on the allowed additive uncertainty. If we wish to work with multiplicative uncertainties, we define relations:

$$L_{mi}(j\omega) = \frac{L_{ai}(j\omega)}{S_i'(\bar{x}_i, j\omega)}, \quad \bar{L}_{mi}(\omega) = \frac{\bar{L}_{ai}(\omega)}{\left| S_i'(\bar{x}_i, j\omega) \right|} \tag{8}$$

3.2 Design of robust controllers

A robust control system for LDS can be designed, for example using the IMC structure (Morari, and Zafiriou, 1989), see Fig. 3. This well-known structure is incorporated into TS block of DPS feedback control system, see Fig. 4.

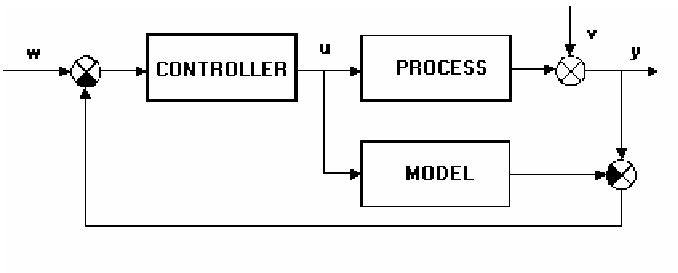

Figure 3: Internal model control structure

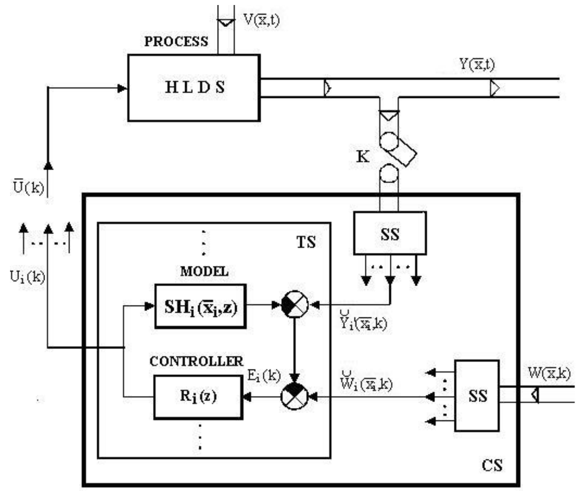

Figure 4: Distributed parameter feedback robust control system

H_2 - optimal controllers $R_i(z)$ are designed by solving the following minimization problem

$$\min_{R_i(z)} \left\| e_i(z) \right\|_2 = \min_{R_i(z)} \left\| \left(1 - SH_i(x_i, z) R_i(z)\right) v_i(z) \right\|_2 \tag{9}$$

subject to the constraint that $R_i(z)$ are stable and causal. Next, controllers $R_i(z)$ are augmented by low-pass filters $F_i(z)$ in the form:

$$F_i(z) = \frac{(1-\alpha_i) z}{z - \alpha_i} \tag{10}$$

where parameters α_i are chosen with respect to accomplish both robust stability and robust performance condition. Then robust controllers in the IMC structure are in the form:

$$\tilde{R}_i(z) = R_i(z) \; F_i(z) \tag{11}$$

4 Modeling of temperature field of glass melting furnace

Melting glass process is typical case of DPS. In the input/output relation it is possible to model it as LDS. Lumped inputs set flow rates of heating medium (earth gas and air) into series of burners located on both sides of the glass furnace above molten glass. Temperature field of the molten glass on the definition domain $\Omega \in E_2$ (in cross-section of the melting space) is distributed output variable, see Fig.5.

Dynamics of a melting glass process as DPS is modelled in the COMSOL Multiphysics software environment, which offers very efficient tool for modelling and simulation scientific and engineering problems based on PDE. Solution of formulated models is realised by FEM.

Is well know, that FEM is a method for solving an equation by approximating continuous quantities as a set of quantities at discrete points, often regularly spaced into a so-called grid or mesh. Because finite element methods can be adapted to problems of great complexity and unusual geometry, they are an extremely powerful tool in the solution of important problems in heat transfer, fluid mechanics, and mechanical systems. Furthermore, the availability of fast and inexpensive computers allows problems which are intractable.

For the recuperative gas glass tank furnace with the cross flame was made two-dimensional space model of the melting glass dynamics based on FEM. In the definition domain is supposed glass melting in steady-state and heating by the mixture of gas and air, which enter into four pairs of burners. Distribution of temperatures in molten glass in the definition domain $\Omega \in \mathrm{E}_2$ is modelled by PDE of parabolic type

$$d\frac{\partial Y}{\partial t} - \nabla(c\nabla Y) + aY = f \tag{12}$$

with constants d, c, a and Neumann type boundary conditions:

$$\vec{n}\left(c\nabla Y\right)+qY=g \tag{13}$$

where $\vec{n}$ is the outward unit normal and q = 0, g = 0.

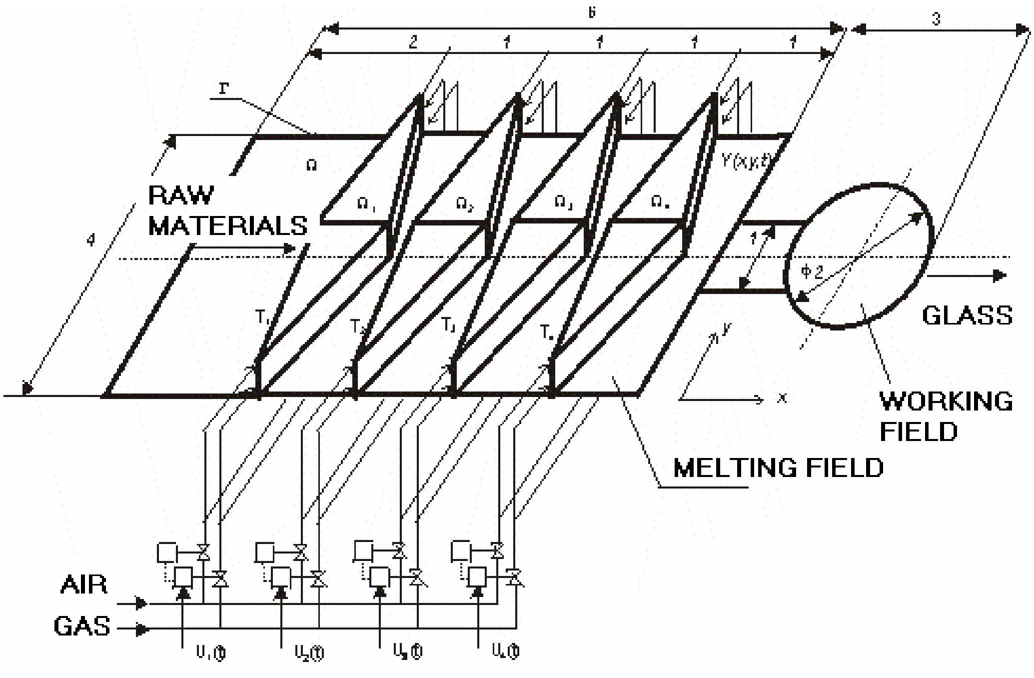

Figure 5: Scheme of recuperative gas glass tank furnace

Modeling process in COMSOL Multiphysics is started via Model Navigator, which enables to select Application Modes. In this case Heat Transfer and Space dimension – 2D was chosen and GUI was activated. There through the toolbar menu both definition domain $\Omega \in E_2$ and subdomains were drawn, see Fig. 6, further in Physics menu parameters of equation system were defined. For this model distributed transient responses from each input actuated on subdomains were obtained, see Fig. 7.

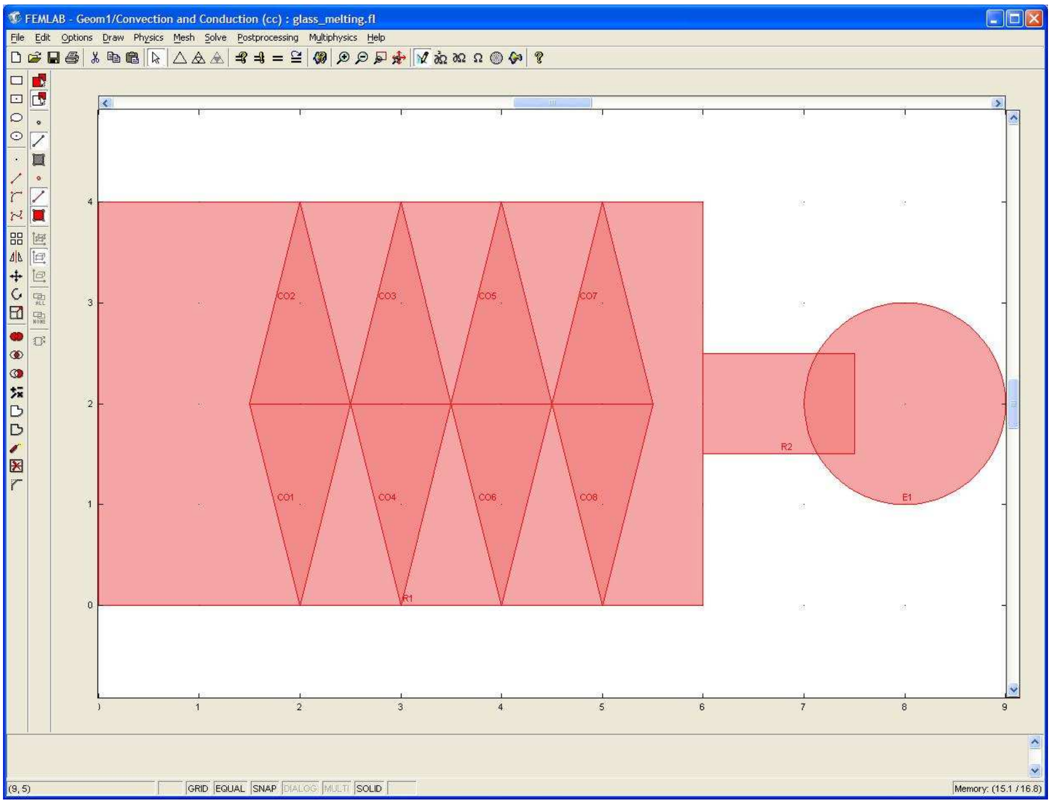

Figure 6: Definition domain

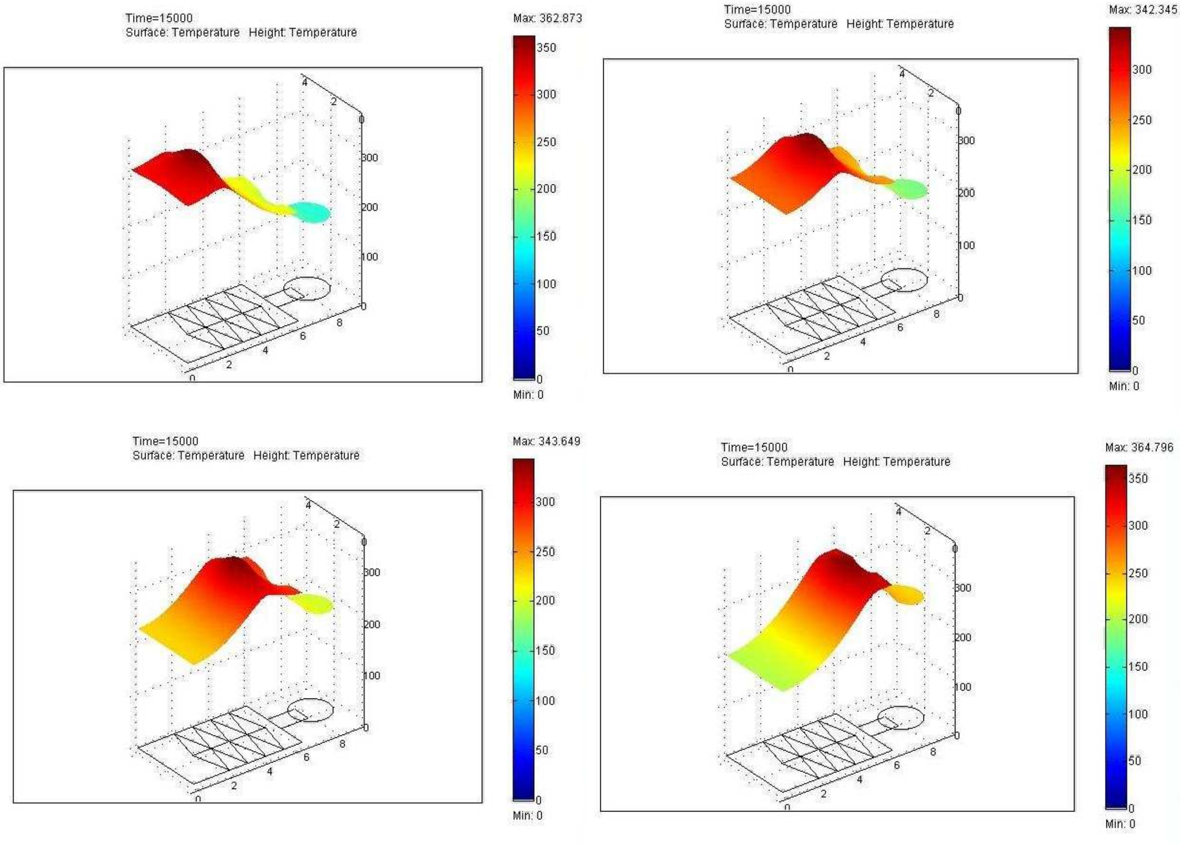

Figure 7: Distributed transient response in steady-state from each lumped input

On distributed transient step responses in steady-state, point with maximal value was determined. Partial distributed transient response in this point was identified, see Fig.8 and discrete transfer function (1) was obtained for i=1, 4 inputs. These transfer function are used for time synthesis. Reduced transient step responses in steady-state (2) for space synthesis have been also expressed, see Fig. 9.

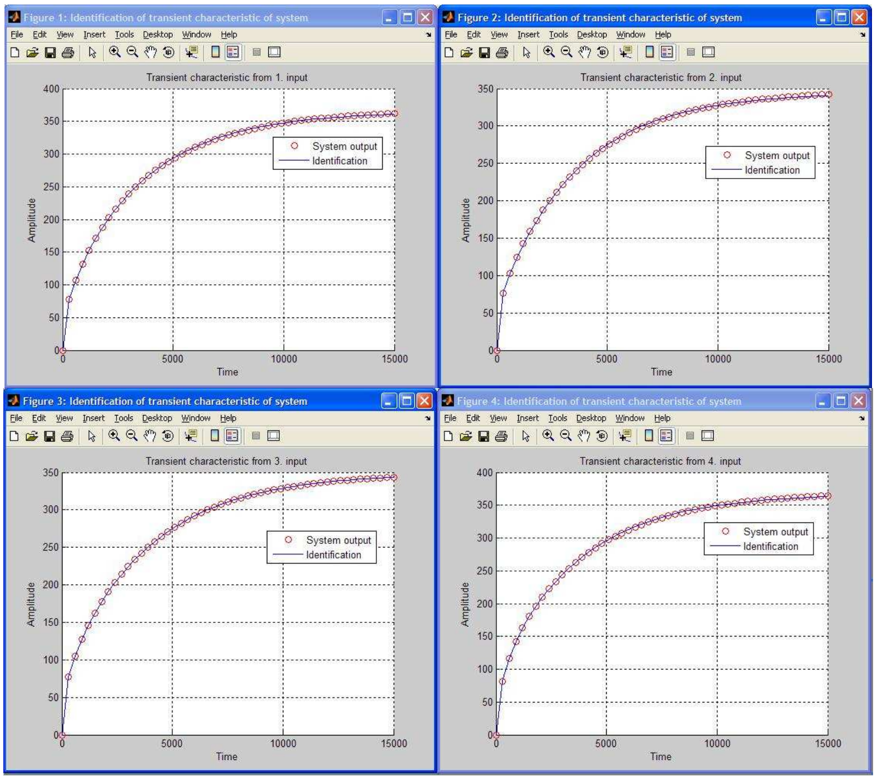

Figure 8: Identification of partial distributed transient responses from each lumped input

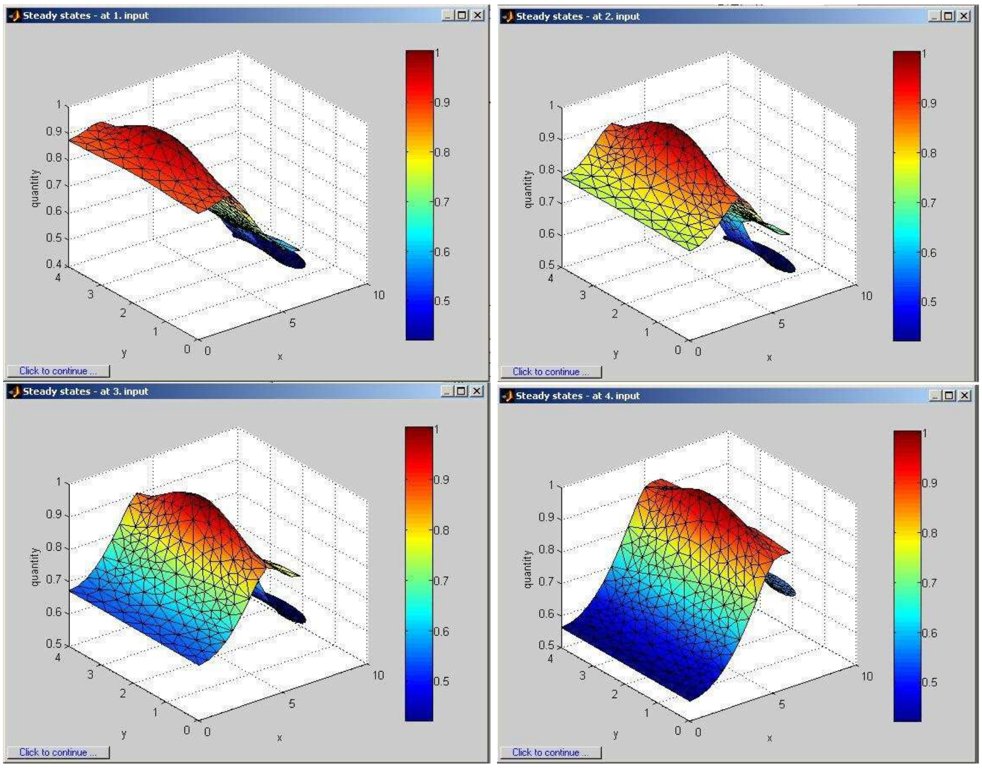

Figure 9: Reduced distributed transient responses in steady-state from each input

5 Distributed Parameter Systems Blockset for MATLAB & Simulink

The Distributed Parameter Systems Blockset is a blockset for use with MATLAB & Simulink for distributed parameter systems and their applications in modeling, control and design of dynamical systems given on complex 3D domains of definition, see Fig. 10, or web side www.dpscontrol.sk .

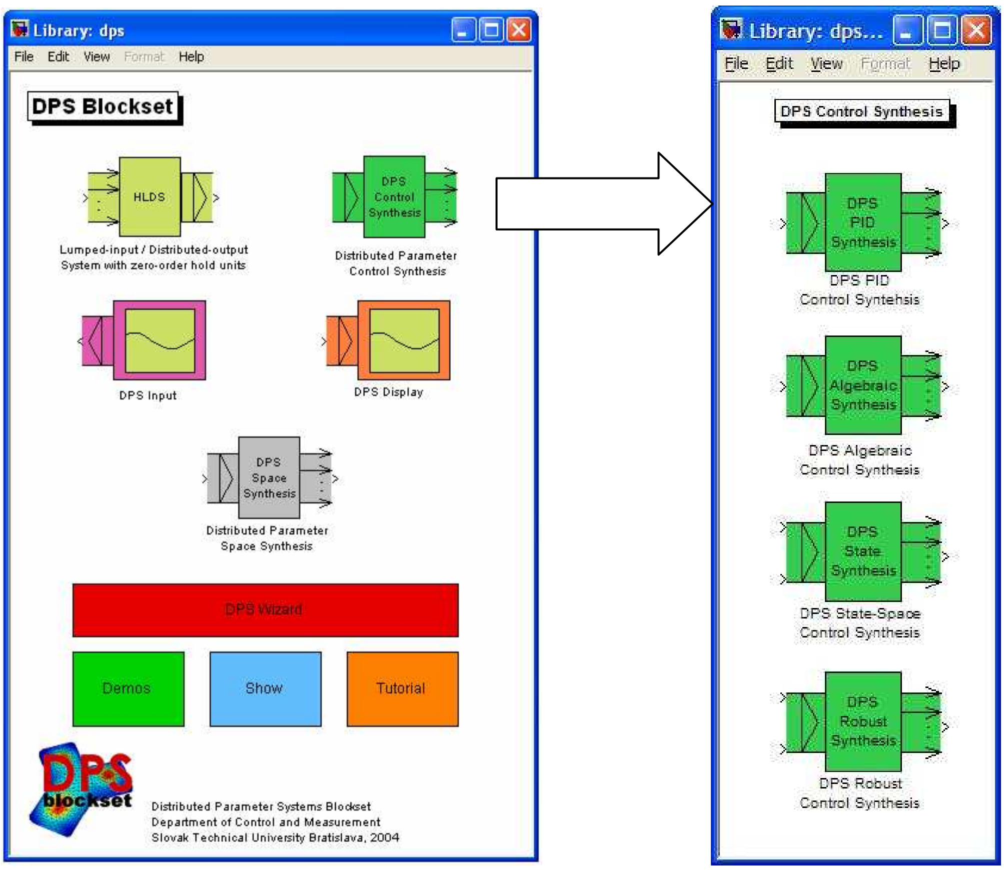

Figure 10: Library of DPS Blockset for MATLAB & Simulink and DPS Control Synthesis methods

The block **HLDS** models controlled distributed parameter systems as lumped-input/distributed-output systems with zero-order hold units. The **DPS Control Synthesis** provides feedback to distributed parameter controlled systems in control loops with blocks for **PID, algebraic, state space and robust** control. The block **DPS Input** generates distributed quantities which can be used as distributed control variables or distributed disturbances, etc. **DPS Display** presents distributed quantities with many options including export to AVI files. The block **DPS Space Synthesis** performs space synthesis as an approximation problem. The block **Tutorial** presents methodological framework for formulation and solution of distributed parameter systems of control. The block **Show** contains motivation examples: Control of temperature field of 3D metal body, Control of 3D beam of „smart" structure, Adaptive control of glass furnace and Groundwater remediation control. The block **Demos** contains examples oriented to methodology of modeling and control synthesis. The **DPS Wizard** in step-by-step operation, by means of five model examples on 1D-3D with default parameters, gives a guide for arrangement and setting distributed parameter control loops.

6 Simulation of control process

In the MATLAB & Simulink environment by means of the DPS Blockset distributed parameter system of robust control is arranged, see Fig. 11. It is feedback control loop, where block Distributed Parameter control Synthesis includes both time part and space part of control synthesis, see Fig 12. In this case, control system consists of four single parameter control loops, where robust controllers based on IMC structure are used. The block DPS Robust Synthesis contains also menu for tuning parameters α_i set up of robust controllers filters (10), see Fig.13.

Simulation results of control process of temperature field of melting glass are on Figures 11, 12. Control objective is certain temperature required by technology with 10% divergence at most. Glass melting process is long-term process with slow dynamics. It is energy demanding process. Therefore, time of steady state acquirement is not as crucial as fuel consumption. With proper set up of controllers can be achieved considerable cost saving. Here controllers were tuned in order to assure aperiodic running of quadratic norm of distributed control error.

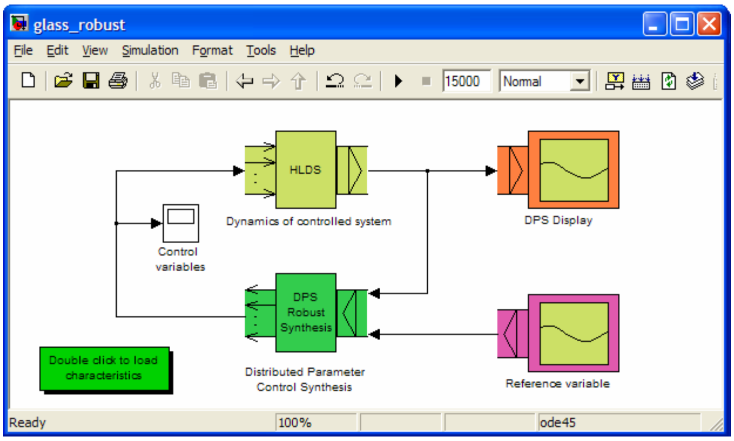

Figure 11: DPS control loop in DPS Blockset for MATLAB & Simulink

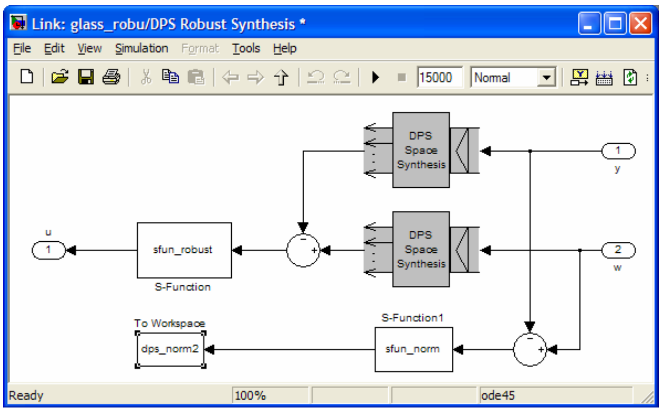

Figure 12: Internal structure of DPS Robust Synthesis block

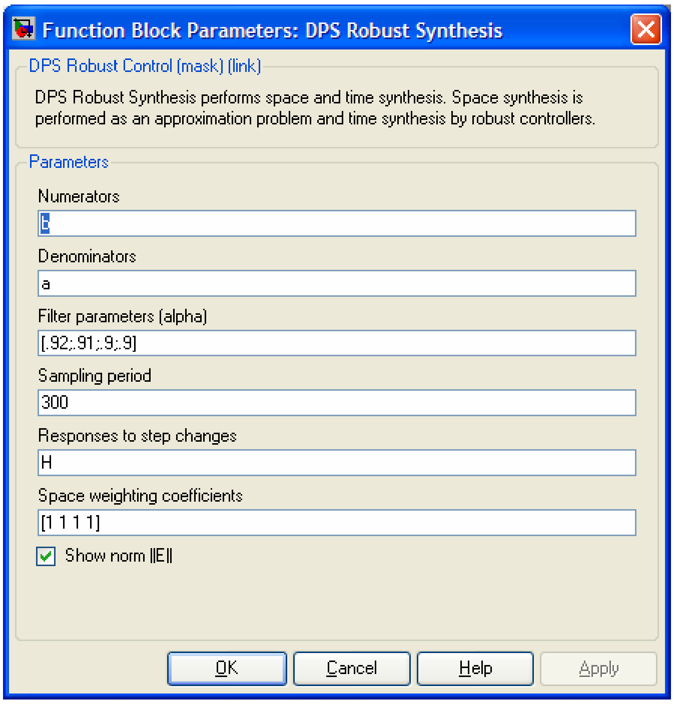

Figure 13: Menu for set up of DPS Robust Synthesis parameters

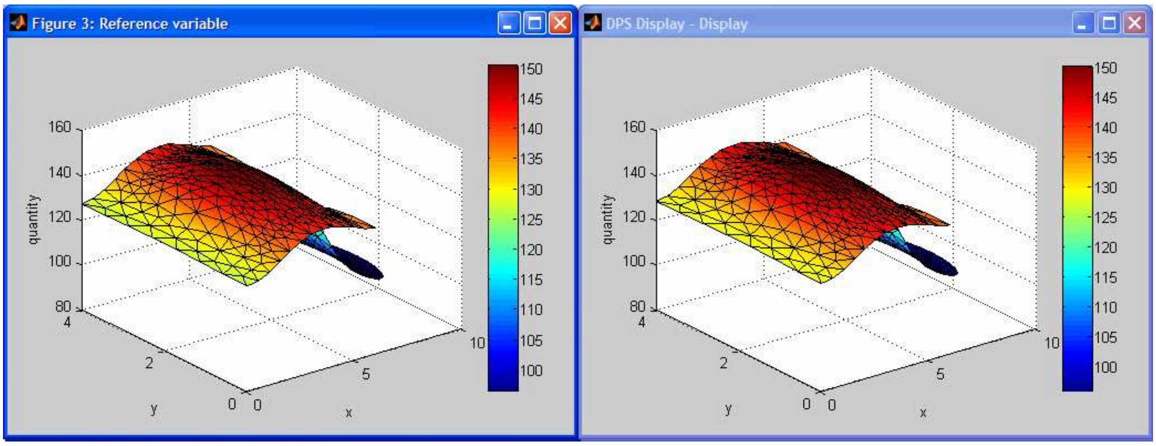

Figure 14: Distributed reference variable W(x,y,t) and controlled variable Y(x,y,t), t→∞

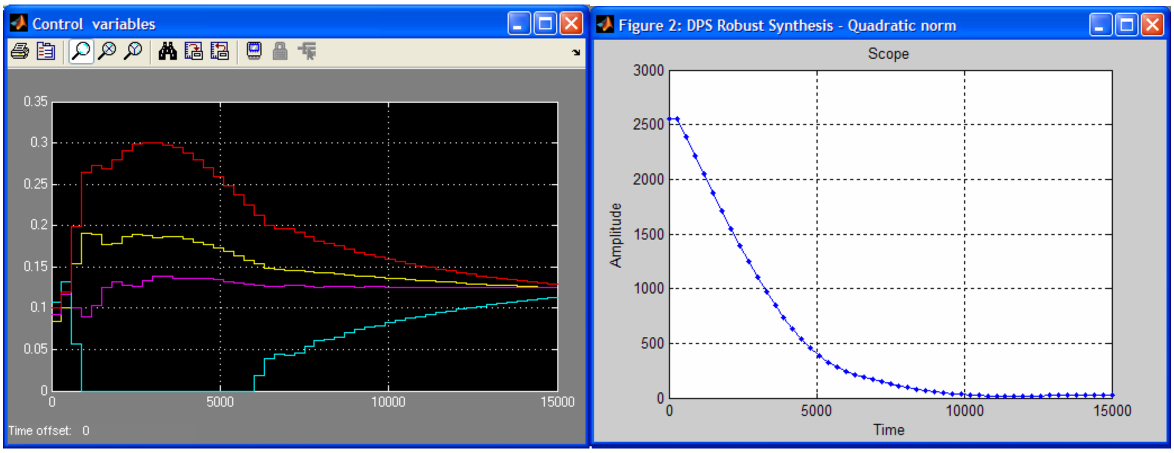

Figure 15: Control variables $U_i(k)$ and quadratic norm of distributed control error ||E(k)||

7 Conclusion

In today's engineering environment huge amount of money are invested into development. Devices have to operate more precisely and react more robustly than ever before. Development of information technologies supports further wide-ranging distribution of diverse methods and software products for 3D numerical dynamical analysis of real systems as distributed parameter systems in any field of technical practice. Temperature field of the melting glass is a typical case of DPS, where in the input/output relation is useful to model it as LDS.

Methodical approach presented in the paper demonstrates simple possibilities, how to exploit of distributed dynamical characteristics, obtained by numerical FEM analysis of systems on complex definition domains for control synthesis of DPS with respect of an uncertainty of models. The DPS Blockset for MATLAB & Simulink provides block-oriented efficient software for this kind of tasks.

ACKNOWLEDGEMENT

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