

EFFICIENT AND ROBUST APPROXIMATION OF MAGNETIZING CHARACTERISTICS

M. Novak, J. Eichler, M. Kosek

Faculty of Mechatronics, Technical University of Liberec, Czech Republic

Abstract

Approximation of amplitude magnetizing characteristic is important both for theory and praxis. Because of the lack of information in literature, several functions with unknown parameters were selected as candidates. The best parameter values were found by the `fminsearch` procedure in MATLAB. From basic functions only hyperbolic tangent with three parameters resulted in acceptable approximation for technical praxis. However, very good approximation was made by the rational function of eight parameters - coefficients. Instead of a graph, eight numbers can be used for the full determination of the characteristic. The coefficients can be transformed into more illustrative zeros and poles in complex plane.

1 Introduction

Irrespective of invasion of digital circuits, the analog devices using ferromagnetic cores are widely used. They are strongly nonlinear and their full description requires measurement of hysteresis loops in steps with increasing excitation. Furthermore, careful experiment with well defined parameters is necessary [1]. From the technical point of view the global characterization, the amplitude characteristics (commutational curve) derived from hysteresis curves, is satisfactory.

The amplitude magnetization characteristic is important from many reasons: in theory for verification of models, in material science for the study of the effects of ferromagnetic material composition, in praxis for comparing cores from different materials and, first of all, in the design of ferromagnetic devices. Therefore the perfect description of amplitude characteristic is very important. On the other hand, because of strong nonlinearity, its perfect description is difficult.

From the theoretical side the universal Preisach model [2] exists. However, its use is not suitable for technical praxis, since it needs deep theoretical understanding the theory and non-standard computations. The good practical way is to use experimental results and approximate them by curves with small number of parameters and minimum possible deviations. To our knowledge, this theme was not treated systematically in the literature. Only several attempts were found [3], [4]. Therefore, in the paper we try to investigate several ways of amplitude characteristic approximation, compare them one another and search for the best solution. We limit to simple approximations, easily made by standard mathematical knowledge and programming in MATLAB. Irrespective of this limitation, good results can be achieved.

In the theoretical part, the paper describes the systematical approach, gives basic formulae and outlines the solution by MATLAB procedures. Although the experiment control is interesting from MATLAB view, only basic results are presented. The comparison of experiment and different ways of approximation, presented namely by a lot of graphs, is in the chapter Results. The main criterion is least squares and according to this criterion the methods are compared in discussion.

2 Theory

The experimental points to be approximated are in Fig. 1 later. The approximation can be made by several ways:

1. Analytical methods that automatically find the best fit between experiment and approximating curve.
2. Basic mathematical functions of shape similar to the experiment.

3. Sophisticated methods used in other scientific areas.

All the methods are described and basic formulae are presented, if necessary. Also the MATLAB processing is explained in some cases for better understanding.

As the criterion of approximation least squares are used. It is a sum of squares of difference between the experiment and approximation in all the experimental points. This simple criterion says, that the lower least squares value, the better approximation.

2.1 Analytical methods

Basic analytical methods are polynomial regression and Fourier transform, applied as the discrete Fourier transform (DFT) or the fast Fourier transform (FFT). They are used very frequently; therefore, both of them are standard MATLAB procedures.

In the polynomial regression the degree of approximating polynomial is selected. However, DFT requires equidistantly spaced experimental points, which is not satisfied, see Fig. 1. It is overcome by using the following procedure. By the application of cubic interpolation any number of points is got. These points are considered as experimental ones together with original ones, although some deviation exists. Since there can be any number of interpolated points, the selection of equidistant ones is not a problem.

Output of DFT or FFT is spectral lines, which are complex, in general. For the approximation spectral lines with highest amplitudes are selected, other lines are set to zero. By the application of inverse DFT to the reduced spectrum, the approximation of experiment is made.

2.2 Basic Mathematical Functions

In general, the following formula can be used for approximating analytical function denoted symbolically as $f(x)$

$$y(x) = Af(k(x - x_0)), \quad (1)$$

where A is the amplitude, k is the coefficient and x_0 is the shift. Amplitude determines the height of function (in the vertical direction for the dependent variable y), coefficient sets up the function width (in horizontal direction, independent variable x) and shift moves the function in the right or left direction (negative or positive shift value, respectively). These three parameters (amplitude, coefficient and shift) should be found for the best possible approximation in the sense of least squares, for instance.

Three approximation functions are considered:

1. Cyclometric function arctan

$$y_c(x) = A \arctan(k(x - x_0)). \quad (2)$$

2. Exponential function

$$y_e(x) = \operatorname{sgn}(x)A(1 - \exp(-k|x - x_0|)). \quad (3)$$

3. Hyperbolic function tanh

$$y_h(x) = A \tanh(k(x - x_0)). \quad (4)$$

In general no procedure for automated finding of the optimum parameters (amplitude A , coefficient k and shift x_0) in the above formulae from (2) to (4) exists. A method of trials and errors must be used. To apply this method handy is a very time consuming task, since three values should be changed.

Fortunately, MATLAB exhibits very useful procedure *fminsearch* that helps to find the coefficients. This procedure changes the parameters (amplitude A , coefficient k and shift x_0 in our case) in order to get minimum of least squares, for instance. The result depends on initial values of these parameters, since there are many local minimums. In order to get the absolute minimum and optimum parameters,

the procedure *fminsearch* should run repeatedly with different initial values. The good way is to use random values of parameters at each run and evaluate the least squares values or graphs, which can be better in many cases.

2.3 Method from Filter Design

In this area the rational function is used in the design of electrical filter. The rational function has the form in general

$$y_k(x) = A \frac{x^N + a_{N-1}x^{N-1} + \dots + a_1x}{x^N + b_{N-1}x^{N-1} + \dots + b_1x + b_0}, \quad (5)$$

where A is multiplying factor, a_k and b_k are coefficients to be found. Both the numerator and denominator polynomials are of the same degree. Absolute member in the numerator is omitted in our case, since the zero value is expected for the zero value of the independent variable x (in the origin). All the parameters (multiplying factor and coefficients) must be found by the application of procedure *fminsearch* again.

By the use of polynomial roots in both the numerator and denominator, the formula (5) can be rewritten in a better descriptive form

$$\hat{y}_z(\hat{x}) = A \frac{(\hat{x} - \hat{z}_1)(\hat{x} - \hat{z}_2) \dots (\hat{x} - \hat{z}_M)}{(\hat{x} - \hat{p}_1)(\hat{x} - \hat{p}_2) \dots (\hat{x} - \hat{p}_M)}, \quad (6)$$

where A is a multiplying factor again, and symbols \hat{z}_k are termed zeros, while for \hat{p}_k the term poles is used. With an exception of multiplying factor A , all items in formula (6) are complex. At zeros the function (6) has zero values and at poles reaches the infinity. Zeros and poles lie in complex plane and are in a conjugate pairs. For our purposes only the real function values for real positive values of independent variable are important.

Again, the position of zeros and poles should be found by the application of procedure *fminsearch*. The advantage of this approach is the geometrical meaning of parameters. By movement of selected pair of zeros or poles, their effect of the function shape can be investigated. Therefore, this approach has tutorial meaning.

3 Experiment

The fully automated experiment controlled by MATLAB script is described elsewhere [1]. Therefore, we present only the amplitude magnetization characteristic in Fig. 1. The characteristic is the dependence of the amplitude of magnetic flux density B on the amplitude of exiting magnetic field strength H . The exciting field is harmonic. The full experimental curve is in Fig. 1a and the most important initial part is in Fig. 1b.

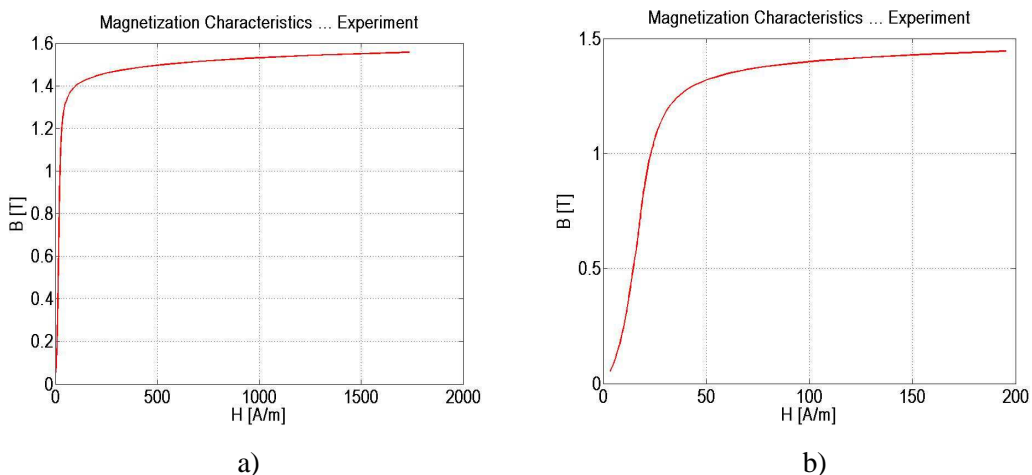


Figure 1: Experimental results. a) Full measurement, b) Initial part.

All the approximating methods were applied to this initial part in order the details to be visible. The curve does not start in the coordinate beginning, since the data file was created automatically from files containing experimental results.

4 Results

Application of MATLAB procedures was described in section Theory. All the possibilities were applied to the experimental points in Fig. 1b. Interesting results will be reported in this section. The organization of this section was taken from the section Theory.

4.1 Analytical methods

Application of polynomial regression leads to results shown in Fig. 2. In Fig. 2a there is an approximation by polynomial of degree 4 and in Fig. 2b is comparison of approximations for several polynomials. The approximation is not good, furthermore for polynomial of degree higher than 4 MATLAB exhibit warnings. The MATLAB recommendations cannot be used in our case. The deviations are high namely for very low magnetic field (impossible negative values) and then in the area of saturation, where the oscillations take a place. Their amplitude increases with increasing polynomial degree. The least squares value in Fig. 2b decreases with increasing polynomial degree, but visually no approximation improvement is visible.

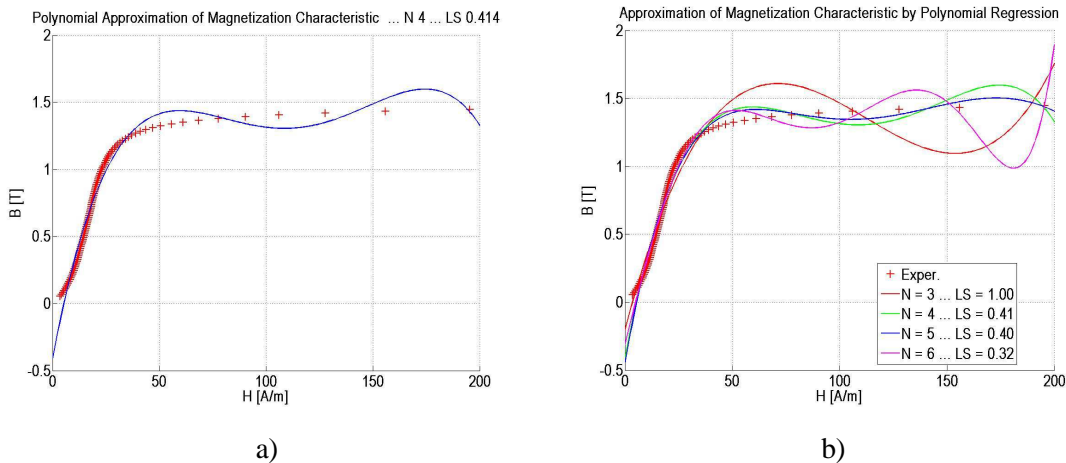


Figure 2: Application of polynomial regression. a) Polynomial of degree 4. b) Effect of different polynomial degree

The examples of DFT applications are in Fig. 3. Fig. 3a shows the effect of 10 spectral lines, while in Fig. 3b there is the effect of increasing number of spectral lines. The approximation is good in the central part, but at the beginning and the end big deviations appear. Their amplitude does not decrease with increasing number of spectral lines, but their number increases. This effect for rectangular signal is known as Gibbs phenomenon.

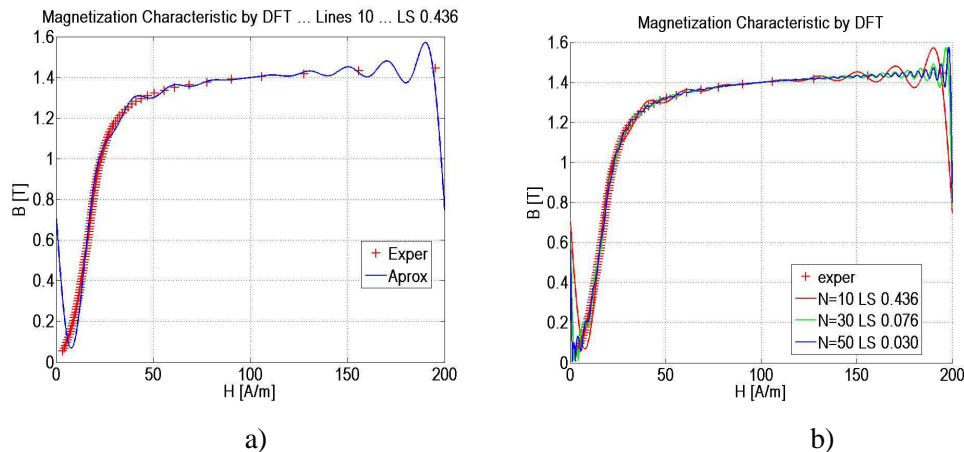


Figure 3: Application of Fourier transform. a) Approximation by 10 spectral lines. b) Effect of different number of spectral lines.

4.2 Basic Mathematical Functions

Basic mathematical functions proposed for approximation are listed in the chapter Theory, formula (2) to (4). Their graphs are in Fig. 4a for a wide range of independent variable and in Fig. 4b for practical use – positive values. Cyclometric function arctan was multiplied by $2/\pi$ in order to converge to 1 at infinity as other two functions. Function tanh has the most rapid change in zero vicinity, while arctan changes most slowly. They form limiting cases.

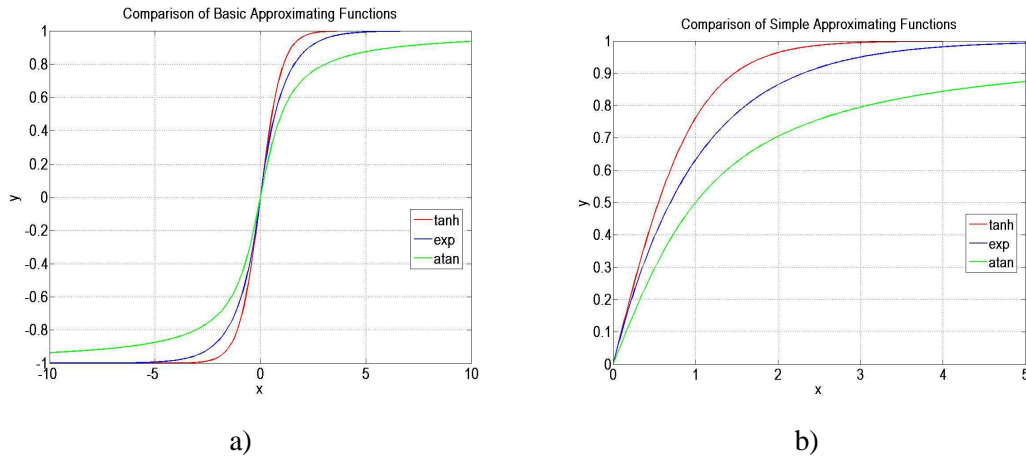


Figure 4: Graphs of basic mathematical functions. a) General. b) Usable for approximation.

These functions can use three or two parameters. Three parameters are necessary for the modeling of hysteresis loop; while two parameters are suitable for the approximation of amplitude characteristics. Nevertheless, we have examined the effect of both three and two parameters in our approximation.

Further, results of approximation for two limiting cases, cyclometric function arctan and hyperbolic function tanh are presented. The approximation made by 3 parameters cyclometric function arctan is in Fig. 5a. Comparison of 3 and 2 parameter approximation is in Fig. 5b. In both the cases the approximation is rough. The deviations for 2 parameters approximation are 3 times higher, but the curve goes through the beginning. The use of 3 parameters exhibit lower deviations, but the approximation does not go through the zero point and has negative values for low excitation.

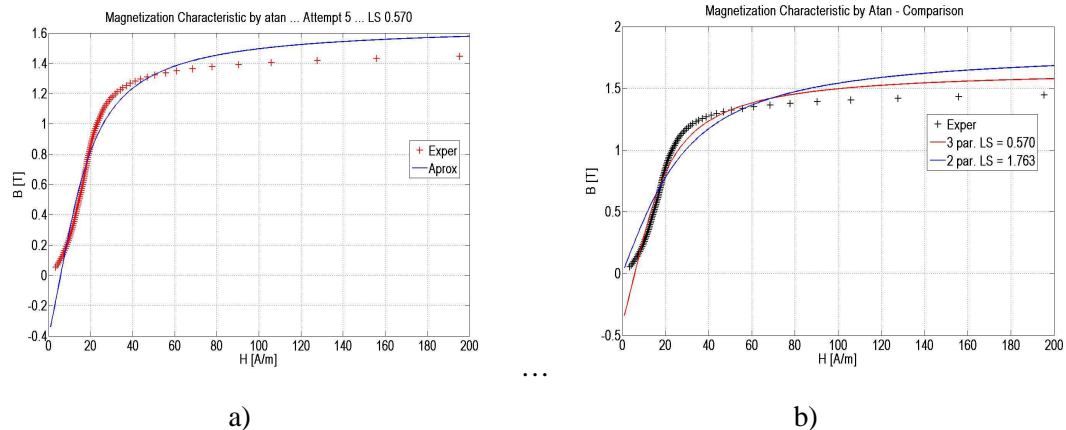


Figure 5: Approximation by the basic mathematical function arctan. a) Use of 3 parameters. b) Comparison of the effect of 2 and 3 parameters.

Results for 3 parameter hyperbolic function tanh are in Fig 6a. The agreement is relatively good with the exception of small values, where negative approximation exists. Least squares confirm the acceptable approximation. The comparison of 2 parameters and 3 parameters approximation is presented in Fig. 6b. The 2 parameters approximation is not good, its deviations are 4 times higher, but is physically acceptable.

4.3 Method of Rational Function

Rational function defined by formula (5) was tested for power 4 ($N = 4$). Therefore, 8 parameters should be found. Results got by fminsearch procedure are shown in Fig. 7. Acceptable, but not perfect, approximation is in Fig. 7a. Some deviations are visible. Optimum result, relatively rare, is in Fig 7b. In this case the deviations are due to the experimental errors probably.

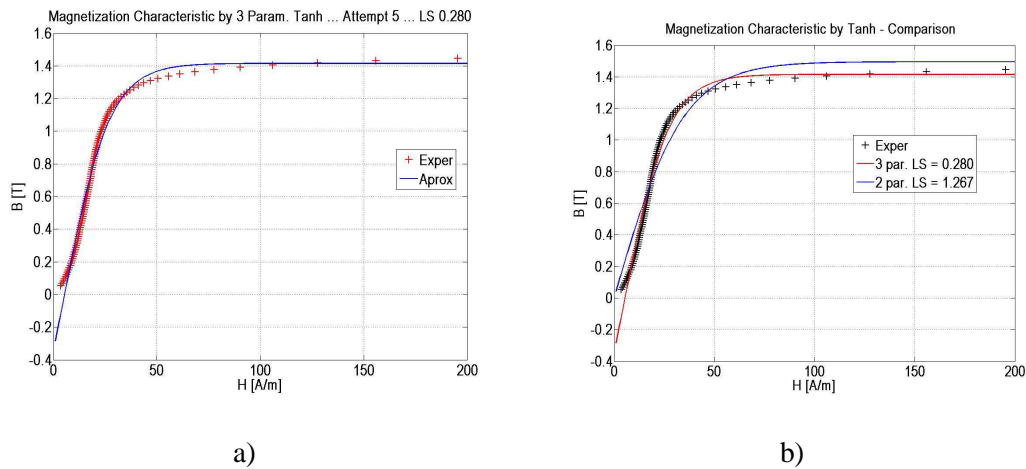


Figure 6: Approximation by the basic mathematical function tanh. a) 3 parameters. b) 2 parameters

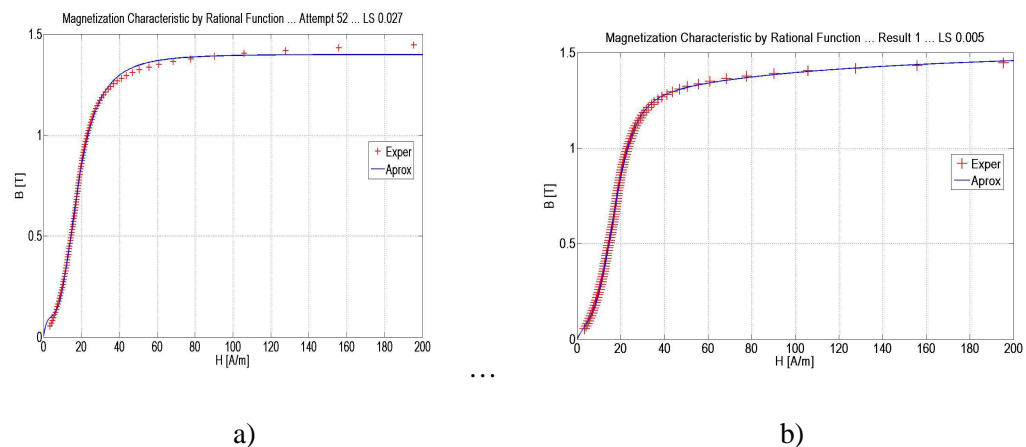


Figure 7: Approximation by the rational function. a) Typical case. b) Optimum result.

Position of zeros and poles for the best approximation in Fig. 7b is in Fig. 8a. One zero and one pole is at the real axis. Remaining two poles and two zeros are complex conjugate. Characteristic reconstructed from pole and zero positions (and multiplying coefficient A in formula (6)) is in Fig. 8b. It is the same as in Fig 7b and uses seven parameters, different in the nature. Approximation method based on finding the optimum zeros and poles position is dual and can be processed similarly.

5 Discussion

Three groups of approximation functions were examined. The first group has the common feature that the approximation is automatic. But the results are not acceptable. The probably explanation is the unusual shape of the approximating function in polynomial regression. This statement is general and it does not mean that in special cases the regression cannot be used efficiently – see Ref. [5], for instance.

The application of DFT is applicable to the periodic functions. The characteristic is treated as one period. Therefore, at the starting and finishing part, there are deviations, since the periodic function must be continuous. In the central part of the period the agreement is good.

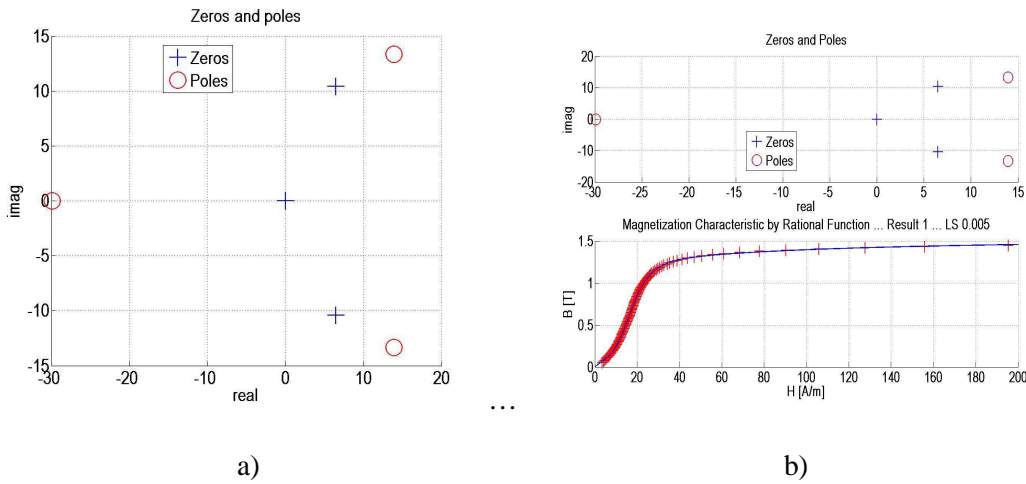


Figure 8: Zeros and poles. a) Position for the best approximation. b) Characteristic derived from the zeros and poles position.

Application of functions in other two groups needs the use of MATLAB procedure for finding the optimum parameters. This procedure is not straightforward and it is possible that the best result will not be found. To our experience by repeating the procedure many times the optimum is found. It can be confirmed by the lowest value of least squares and almost the same values of parameters reached by repeated finding.

The difference between last two groups is that the functions in the second group are simple and have maximum three parameters. Low number of parameter results in a low quality of approximation. Only hyperbolic tangent is acceptable. Again, it does not mean that in special cases other two functions cannot give good results. We have not considered other similar well-known basic functions, as logarithm or general power. One of reasons was their behavior at low values of independent variable.

The only success was for the rational function of the third group. The agreement with experiment was almost perfect. We applied it in the coefficient form only, but the same results can be achieved by the use of zeros and poles. Furthermore, the second approach is better for understanding and exhibit new possibilities. One from explanations of the good approximation is higher number of parameters, 8 for coefficients and 7 for zeros and poles. On the other hand, the computation time for rational function is many times higher than for the basic functions. This can be explained by many unknown parameters. The acceleration can be made by parallel computing.

Comparison of known and estimated parameters for some of examined functions is in Tab. 1. The least squares (LS) value is the lowest value that can be achieved. The success means the relative number of iteration that resulted in given least squares value. In the case of rational function only two iterations from 100 were successful. For lower LS values the success is zero in all the cases.

Table 1: COMPARISON OF SOME APPROXIMATION FUNCTIONS

Function	Parameters	LS	Success [%]	Time [ms]
Arctan	3	0.57	95	3
Arctan	2	1.76	98	3
Tanh	3	0.28	75	5
Tanh	2	1.26	80	5
Rational	8	0.005	2	300

The success values are estimated, not measured carefully, although it is possible and relatively simple. Time of iteration is estimated too and very roughly, although it can be measured in principle. It follows from Tab. 1 that rational function results in two order better approximation; its success is about two orders lower and computational time two order higher (very rough estimation).

The criterion of approximation quality was sum of squares of deviations termed least squares. This criterion works well; nevertheless graphical output is necessary for efficient check, since the deviations can be concentrated in some areas. An example is the 3 parameter basic functions. The deviations are smaller than for 2 parameters, but the approximation is not acceptable for low values of independent variable (negative values).

Usually the good approximation for rational function is made in several minutes. If the sum of squares is under the given value, the graph appears and the user can decide if the result is acceptable. If yes, the parameters (coefficients) are saved in the file for later use.

6 Conclusion

The approximation of amplitude magnetization characteristic was studied systematically and, may be, in detail with these results. No standard automated approximation procedure is acceptable in general. It can be explained by unusual shape of characteristic. It means that semiautomatic solution is necessary. Suitable function is selected with unknown parameters and their values for the best fit are found by MATLaB procedure.

From the basic functions only the hyperbolic tangent can be used, if the requirements for quality are not strong. On the other hand, very good results can be achieved by rational function. The rational function can be done in complex plane by zeros and poles. The best fit requires finding their positions. This approach, widely used in electronics, is illustrative.

Success of approximation depends on the number of parameters among other things. Three parameters are not enough, eight parameters ensures very good approximation. For the praxis it is important that the whole characteristic is given by eight numbers.

We did not make extensive literature research. To our knowledge we did not meet such approach, so we can expect that the results are new.

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Miroslav Novák

Faculty of mechatronics, TU Liberec, Studentská 2, 461 17 Liberec, email: mirosav.novak@tul.cz

Jakub Eichler

Faculty of mechatronics, TU Liberec, Studentská 2, 461 17 Liberec, email: jakub.eichler@tul.cz

Miloslav Košek

Faculty of mechatronics, TU Liberec, Studentská 2, 461 17 Liberec, email: miloslav.kosek@tul.cz