# DC ANALYSIS OF CIRCUITS CONTAINING OPERATIONAL AMPLIFIERS IN MATLAB

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**Abstract**

**DC analysis of circuits containing operational amplifiers is sometimes time-consuming activity also in case when the operational amplifiers are analyzed as black boxes. It means that only terminal behavior of operational amplifiers is taken into account and it is not attended to the internal characteristic of the operational amplifiers. Using symbolic and numeric computation of MATLAB together with the sparse tableau analysis saves a lot of time and effort because formulation of the circuit equations is done in systematic and automatic way for every circuit containing operational amplifiers and there is no difficulty in writing as well as solving circuits with operational amplifiers.**

## 1 Principles of DC Analysis of Electric Circuits Containing Operational Amplifiers

DC analysis of electric circuits containing operational amplifiers implies problem of solving a set of algebraic circuit equations in order to find all branch currents and voltages. To obtain a complete description of a general circuit, the set of circuit equations is needed. This set consists of the equations depending on the topology of the circuit and the equations depending on the type of the circuit elements. The equations, depending on the topology of the circuit, represent how the circuit elements are connected to one another in the circuit. These equations are called the connection equations. The equations, which depend on the type of the circuit element, describe the voltage-current relationship and they are called the element equations or branch equations. If the voltage-current characteristics for two-terminal elements and two-port elements in the circuit are linear, and time-invariant, then the set of the equations describing the circuit, is also linear.

The best way for circuit equations formulation is to use a method that enables systematic and automatic formulation of the circuit equations for every circuit as well as for circuits containing operational amplifiers. For such purposes, two methods are appropriate: modified nodal analysis and sparse tableau analysis. Sparse tableau analysis is most general formulation of the equations describing the circuit because the solution provides the currents through all elements, the voltages across all elements and all nodal voltages simultaneously. This method consists of the connection equations and the element equations.

The connection equations are obtained by applying Kirchhoff's laws to the circuit, which leads to the two sets of linear algebraic equations in terms of the branch currents and the branch voltages. The set of connection equations must be linearly independent.

The first set of the connection equations can be expressed [1], [3]:

$$\mathbf{A}\,\mathbf{i} = \mathbf{0}\,, \tag{1}$$

where $\mathbf{i}$ is a matrix of branch currents, $\mathbf{A}$ being the node versus branch reduced-incidence matrix.

The second set of the connection equations can be expressed in terms of the branch voltages using the fundamental loop versus branch incidence matrix. The better way is to convey this set of the equations in terms of the branch voltages and the node voltages as [3]:

$$\mathbf{u} = \mathbf{A}^{\mathrm{T}}\,\mathbf{v}\,, \tag{2}$$

where $\mathbf{A}^{\mathrm{T}}$ is the transposed matrix $\mathbf{A}$, $\mathbf{u}$ being the branch voltage vector, $\mathbf{v}$ being the node voltage vector.

The element equations are related according to the voltage-current characteristics of the elements. For linear circuits, containing the ideal operational amplifiers too, the element equations can be expressed [3]:

$$\mathbf{K}_\mathrm{u}\mathbf{u} + \mathbf{K}_\mathrm{i}\mathbf{i} = \mathbf{s}\,, \tag{3}$$

where $\mathbf{K}_\mathrm{u}, \mathbf{K}_\mathrm{i}$ are the matrices containing the coefficients that define the linear voltage-current relationships for the circuit elements uniquely, $\mathbf{s}$ being the vector containing the parameters of the independent voltage and current sources.

Assembling the equations (1), (2), and (3), the sparse tableau equations are constituted. It is convenient to rewrite the sparse tableau equations as a single matrix equation [3]:

$$\mathbf{T}\mathbf{x} = \mathbf{w}\,, \tag{4}$$

where $\mathbf{T}$ is the square tableau matrix, $\mathbf{x}$ being the vector of unknown variables, $\mathbf{w}$ being the vector containing zero vectors of appropriate dimensions and the vector $\mathbf{s}$.

The operational amplifier is an electronic circuit element that is designed for using with other circuit elements to perform a specified signal-processing function. In order to solve the circuits containing operational amplifiers, it is necessary to have a model of operational amplifier. Several models of operational amplifiers, of varying accuracy and complexity, are available for operational amplifiers [1]. However, when the black-box approach is chosen, the simplest model of the operational amplifier can be used. This model is called as the ideal operational amplifier and it is shown in Figure 1.

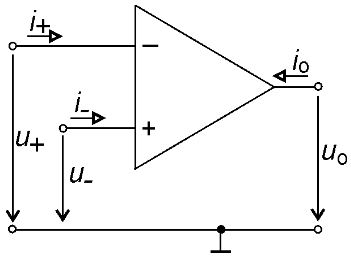



Figure 1: The ideal operational amplifier

For the ideal operational amplifier the following two conditions are valid.

The first condition implies that the input resistance between the two input terminals of the ideal operational amplifier is assumed zero. In this case, the input terminals can be considered as being open in terms of current in the sense that no current flows into or out of them [1]:

$$i_+ = 0 \text{ and } i_- = 0\,, \tag{5}$$

where $i_+, i_-$ are the currents flowing into input positive, negative terminal, respectively.

The second condition implies that the two input terminals of the ideal operational amplifier with a negative feedback path can be considered as being short in terms of voltages in the sense that the voltage levels at the two input terminals are almost equal [1]:

$$u_+ \cong u_-\,, \tag{6}$$

where $u_+, u_-$ are the voltages at input positive, negative terminal, respectively.

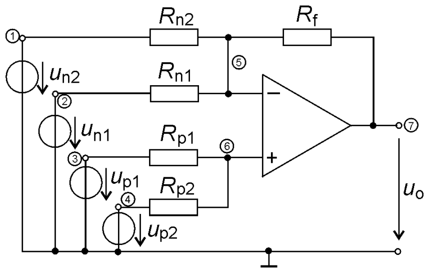
All the above given steps which must be done to solve the circuits containing operational amplifiers can be easily executed using the MATLAB program, especially the symbolic computation.

## 2 Results

The proposed procedure for DC analysis of circuit containing operational amplifiers is applied to the circuit (the general combiner with operational amplifier) shown in Figure 2 (left). In this circuit, the aim is to find the output voltage $u_0$ in terms of the input voltages $u_{\mathrm{n}1}, u_{\mathrm{n}2}, u_{\mathrm{p}1}, u_{\mathrm{p}2}$, respectively.

All the computations for DC analysis of given circuit are done by running MATLAB program based on the sparse tableau analysis. The composed program formulates the tableau equations for an arbitrary circuit containing operational amplifiers automatically and solves them. The solution can be given in symbolic and/or numeric form.

The voltage $u_0$ equals to the node voltage $v_7$ and its expression in analytic form, is shown in Figure 2 (right).

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v 7 =

$$-\frac{rF\ uN1}{rN1}-\frac{rF\ uN2}{rN2}+\frac{rP2\ (rF\ rN2+rF\ rN1+rN1\ rN2)\ uP1}{rN2\ rN1\ (rP1+rP2)}+\frac{rP1\ (rF\ rN2+rF\ rN1+rN1\ rN2)\ uP2}{rN2\ rN1\ (rP1+rP2)}$$

Figure 2: The circuit for DC analysis (left) and analytic (symbolic) expression of the output voltage (right)

The branch currents and the branch voltages are shown in Figure 3.

i 1 =

$$-\frac{uN1}{rN1}+\frac{rP2\ uP1}{rN1\ (rP1+rP2)}+\frac{rP1\ uP2}{rN1\ (rP1+rP2)}$$

i 2 =

$$-\frac{uN2}{rN2}+\frac{rP2\ uP1}{rN2\ (rP1+rP2)}+\frac{rP1\ uP2}{rN2\ (rP1+rP2)}$$

i 3 =

$$-\frac{uP1}{rP1+rP2}+\frac{uP2}{rP1+rP2}$$

i 4 =

$$\frac{uP1}{rP1+rP2}-\frac{uP2}{rP1+rP2}$$

i 5 =

$$\frac{uN1}{rN1}-\frac{rP2\ uP1}{rN1\ (rP1+rP2)}-\frac{rP1\ uP2}{rN1\ (rP1+rP2)}$$

i 6 =

$$\frac{uN2}{rN2}-\frac{rP2\ uP1}{rN2\ (rP1+rP2)}-\frac{rP1\ uP2}{rN2\ (rP1+rP2)}$$

i 7 =

$$\frac{uP1}{rP1+rP2}-\frac{uP2}{rP1+rP2}$$

i 8 =

$$-\frac{uP1}{rP1+rP2}+\frac{uP2}{rP1+rP2}$$

i 9 =

$$-\frac{uN1}{rN1}-\frac{uN2}{rN2}+\frac{rP2\ (rN2+rN1)\ uP1}{rN2\ rN1\ (rP1+rP2)}+\frac{rP1\ (rN2+rN1)\ uP2}{rN2\ rN1\ (rP1+rP2)}$$

i10 =

$$0$$

i11 =

$$\frac{uN1}{rN1}+\frac{uN2}{rN2}-\frac{rP2\ (rN2+rN1)\ uP1}{rN2\ rN1\ (rP1+rP2)}-\frac{rP1\ (rN2+rN1)\ uP2}{rN2\ rN1\ (rP1+rP2)}$$

u 1 =

$$uN1$$

u 2 =

$$uN2$$

u 3 =

$$uP1$$

u 4 =

$$uP2$$

u 5 =

$$uN1-\frac{rP2\ uP1}{rP1+rP2}-\frac{rP1\ uP2}{rP1+rP2}$$

u 6 =

$$uN2-\frac{rP2\ uP1}{rP1+rP2}-\frac{rP1\ uP2}{rP1+rP2}$$

u 7 =

$$\frac{rP1\ uP1}{rP1+rP2}-\frac{rP1\ uP2}{rP1+rP2}$$

u 8 =

$$-\frac{rP2\ uP1}{rP1+rP2}+\frac{rP2\ uP2}{rP1+rP2}$$

u 9 =

$$-\frac{rF\ uN1}{rN1}-\frac{rF\ uN2}{rN2}+\frac{rF\ rP2\ (rN2+rN1)\ uP1}{rN2\ rN1\ (rP1+rP2)}+\frac{rF\ rP1\ (rN2+rN1)\ uP2}{rN2\ rN1\ (rP1+rP2)}$$

u10 =

$$0$$

u11 =

$$-\frac{rF\ uN1}{rN1}-\frac{rF\ uN2}{rN2}+\frac{rP2\ (rF\ rN2+rF\ rN1+rN1\ rN2)\ uP1}{rN2\ rN1\ (rP1+rP2)}+\frac{rP1\ (rF\ rN2+rF\ rN1+rN1\ rN2)\ uP2}{rN2\ rN1\ (rP1+rP2)}$$

Figure 3: The branch currents (left) and the branch voltages (right), both expressed in symbolic form

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